Name:
Period: Date:

8.2 – Test Review

Rectilinear motion

<u>Purpose</u>: Analyze motion of a particle moving in one dimension.

<u>Tips/Method</u>: see textbook p.327

1) Velocity = $\frac{d}{dt}$ Position
2) Acceleration = $\frac{d^2}{dt^2}$ Position
3) "Speeding up" = acceleration and velocity have the same sign.
4) "Slowing down" = acceleration and velocity have opposite signs
5) "Stopped" = Velocity is 0
6) "Changes Direction" = Position reaches an extremum

A particle's position on the x-axis can be modeled by $x(t) = 2t^3 - 21t^2 + 60t + 3$ for $t \geq 0$.

A) Where is the particle when it has stopped?
B) When is the particle speeding up? Slowing down?
C) When does the particle's motion first change direction?
D) When does the particle reach its maximum velocity on $0 \leq t \leq 10$

A) $V(t) = 0$, what is $X(t)$?

$V(t) = 6t^2 - 42t + 60$

$0 = 6(t^2 - 7t + 10)$

$0 = (t-5)(t-2)$

$t = \{2, 5\}$

$\boxed{\begin{array}{l} X(2) = 2(8) - 21(4) + 120 + 3 \\ X(5) = 2(125) - 21(25) + 300 + 3 \end{array}}$

B) $V(t)$ critical values $= \{2, 5\}$

$a(t) = 12t - 42$

$0 = 12t - 42$

$t = \frac{42}{12}$

$t = \frac{7}{2}$

	0 to 2	2 to $\frac{7}{2}$	$\frac{7}{2}$ to 5	5 to 10
$v(t)$	+	−	−	+
$a(t)$	−	−	+	+
	SLOW	SPEED	SLOW	SPEED

C) $\boxed{t=2}$ when $v(t)$ first changes sign

D) $v(t) = 6t^2 - 42t + 60$

MAX on $[0, 10]$

Extreme value thm

$v(0) = 60$

$v(\frac{7}{2}) = -13.5$

$v(10) = 240$

@ $\boxed{t = 10}$

2016-2017 AP Calculus BC
Week 8

Name:
Period: Date:

Mean Value Theorem and Rolle 's Theorem

<table>
<tr>
<td>Theorem (copy):

Purpose: Proves the existence of a point with a specified derivative value. Also, can be used to relate derivatives and function values.</td>
<td>Find all values of x that satisfy the Mean Value Theorem for $f(x) = \sin x$ on $\left[\frac{\pi}{2}, \frac{7\pi}{2}\right]$
$f'(c) = \cos c = \frac{\sin\frac{7\pi}{2} - \sin\frac{\pi}{2}}{7\pi/2 - \pi/2} = \frac{-1-1}{3\pi} = \frac{-2}{3\pi}$ $c = \cos^{-1}\left(\frac{-2}{3\pi}\right)$
Application: An object's vertical height above ground at t seconds is represented by $h(t)$. At 6 seconds, the object is 2 feet below ground. The objects maximum velocity is 10 ft/sec. What is the maximum height that the particle could reach at 15 seconds?
$h(6) = -2$
$h'(x) \leq 10$
$h(15) \leq ?$
some x on [6,15] $h'(x) = \frac{h(15) - h(6)}{15-6}$
$\frac{h(15)+2}{9} \leq 10$
$h(15) \leq 88$
$\boxed{88 \text{ ft}}$</td>
</tr>
</table>

Optimization

<table>
<tr>
<td>Purpose: Maximize or minimize a function given constraints – usually an application problem.

Tips/Method:
1) Identify the quantity you are trying to maximize/minimize.
2) Write a function that models that quantity in terms of one other variable, given the restrictions in the problem.
3) If on a closed interval: Use Extreme Value Theorem: check endpoints and critical values for the absolute extrema
4) If on an open interval: Find critical values and use a sign chart to locate relative extrema. Use end behavior to identify absolute extrema.</td>
<td>What are the dimensions of the largest right cylinder that will fit inside of a right circular cone with volume 100 cm^3?
see photo of board</td>
</tr>
</table>

Name:
Period: Date:

Unit 0 Topics:

Linear Approximation

<u>Purpose:</u> Use a tangent line to approximate the value of a function near the point of tangency

<u>Tips/Method:</u>
1) Select an x-value that can be precisely evaluated.
2) Find the equation of the line tangent to $f(x)$ at that point.
3) Substitute the desired "input" into the tangent line function.

Approximate $\sin(138°)$ $135° = \frac{3\pi}{4}$

@ $\frac{3\pi}{4}$ $f(x) = \sin x$

$\ell(x) = f'(\frac{3\pi}{4})(x - \frac{3\pi}{4}) + f(\frac{3\pi}{4})$

$\ell(x) = \cos(\frac{3\pi}{4}) \cdot (x - \frac{3\pi}{4}) + \sin\frac{3\pi}{4}$

$\ell(x) = -\frac{\sqrt{2}}{2}(x - \frac{3\pi}{4}) + \frac{\sqrt{2}}{2}$

$138° \doteq 2.409$

$\ell(2.409) \doteq \boxed{-\frac{\sqrt{2}}{2}(2.409 - \frac{3\pi}{4}) + \frac{\sqrt{2}}{2}}$

(approx)

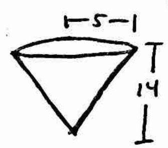
Related Rates

<u>Purpose:</u> Find an instantaneous rate of change, given another related rate of change.

<u>Tips/Method:</u>
1) Identify the given rate and unknown rate.
2) Use algebra/geometry to write an equation relating the two changing quantities, with <u>no other variables</u>.
3) Implicit differentiation with respect to time.
4) Substitute in all given information and solve for the unknown rate.

A conical tank of water is leaking water at a constant rate of 2 cm^3/hr. The tank is 10 cm across and 14 ft tall. At what rate is the radius of the top of the water in the tank changing when the depth of the water is 6 cm?

GIVEN $\frac{dV}{dt} = -2$ WANT $\frac{dR}{dt}$



$\frac{R}{h} = \frac{5}{14}$

$5h = 14R$

$h = \frac{14}{5}R$

EQN w/ V & R:

$V = \frac{\pi}{3}R^2 h$

$V = \frac{\pi}{3} \cdot \frac{14}{5} R^3$

$V = \frac{14\pi}{15} R^3$

when $h = 6$,

$R = 6(\frac{5}{14}) = \frac{30}{14}$

$\frac{dV}{dt} = \frac{14\pi}{15} \cdot 3R^2 \frac{dR}{dt}$

$-2 = \frac{14\pi}{15} \cdot 3 \cdot (\frac{30}{14})^2 \frac{dR}{dt}$

$\frac{dR}{dt} =$

Derivative of an Inverse Function

<u>Purpose:</u> Find the derivative of an inverse function without actually finding the inverse function

<u>Tips/Method:</u>
1) Verify that the function has an inverse by checking that it is <u>one-to-one</u>
2) If f and g are inverses:

$$f(g(x)) = x$$
$$f'(g(x)) \cdot g'(x) = 1$$
$$g'(x) = \frac{1}{f'(g(x))}$$

A) Let y = f(x). Find $\frac{d}{dx}f^{-1}(2)$ if $xy = y + 4$

Inverse: $yx = x + 4$ $\rightarrow$ $y = \frac{x+4}{x}$

$y(2) = \frac{6}{2} = 3$

$\frac{d}{dx}$: $y'x + y = \cancel{x} + 4$

$x=2$: $y'2 + y = \cancel{2} + 4$

$y' = \frac{5-y}{2}$

$\boxed{y' = \frac{2}{2}} = \boxed{-1}$

B) f and g are inverse functions. Find g'(2)

x	0	1	2	3	4
f(x)	-5	8	1	2	4
f'(x)	7	9	-3	8	0

$f(3) = 2$
$g(2) = 3$

$g'(x) = \frac{1}{f'(g(x))}$

$g'(2) = \frac{1}{f'(g(2))}$

$= \frac{1}{f'(3)}$

$= \frac{1}{8}$

Extra Practice: Chapter 5 Supplementary Exercises #1, 5-16, 20-23, 25, 26, 28, 29, 34, 36, 39, 40, 45, 46, 47, 48