

Review #8 scoring guidelines

AP[®] CALCULUS AB 2004 SCORING GUIDELINES (Form B)

Question 2

For $0 \leq t \leq 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t} \cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

- Show that the number of mosquitoes is increasing at time $t = 6$.
- At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.
- According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.
- To the nearest whole number, what is the maximum number of mosquitoes for $0 \leq t \leq 31$? Show the analysis that leads to your conclusion.

(a) Since $R(6) = 4.438 > 0$, the number of mosquitoes is increasing at $t = 6$.

1 : shows that $R(6) > 0$

(b) $R'(6) = -1.913$
Since $R'(6) < 0$, the number of mosquitoes is increasing at a decreasing rate at $t = 6$.

2 : $\left\{ \begin{array}{l} 1 : \text{considers } R'(6) \\ 1 : \text{answer with reason} \end{array} \right.$

(c) $1000 + \int_0^{31} R(t) dt = 964.335$
To the nearest whole number, there are 964 mosquitoes.

2 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

(d) $R(t) = 0$ when $t = 0$, $t = 2.5\pi$, or $t = 7.5\pi$
 $R(t) > 0$ on $0 < t < 2.5\pi$
 $R(t) < 0$ on $2.5\pi < t < 7.5\pi$
 $R(t) > 0$ on $7.5\pi < t < 31$
The absolute maximum number of mosquitoes occurs at $t = 2.5\pi$ or at $t = 31$.

$$1000 + \int_0^{2.5\pi} R(t) dt = 1039.357,$$

There are 964 mosquitoes at $t = 31$, so the maximum number of mosquitoes is 1039, to the nearest whole number.

4 : $\left\{ \begin{array}{l} 2 : \text{absolute maximum value} \\ 1 : \text{integral} \\ 1 : \text{answer} \\ 2 : \text{analysis} \\ 1 : \text{computes interior critical points} \\ 1 : \text{completes analysis} \end{array} \right.$

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \leq t \leq 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

- (a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?
- (b) How many gallons of water are in the tank at time $t = 3$ minutes?
- (c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t .
- (d) At what time t , for $0 \leq t \leq 120$, is the amount of water in the tank a maximum? Justify your answer.

(a) Method 1: $\int_0^3 \sqrt{t+1} dt = \frac{2}{3}(t+1)^{3/2} \Big|_0^3 = \frac{14}{3}$

- or -

Method 2: $L(t)$ = gallons leaked in first t minutes

$$\frac{dL}{dt} = \sqrt{t+1}; \quad L(t) = \frac{2}{3}(t+1)^{3/2} + C$$

$$L(0) = 0; \quad C = -\frac{2}{3}$$

$$L(t) = \frac{2}{3}(t+1)^{3/2} - \frac{2}{3}; \quad L(3) = \frac{14}{3}$$

(b) $30 + 8 \cdot 3 - \frac{14}{3} = \frac{148}{3}$

(c) Method 1:

$$\begin{aligned} A(t) &= 30 + \int_0^t (8 - \sqrt{x+1}) dx \\ &= 30 + 8t - \int_0^t \sqrt{x+1} dx \end{aligned}$$

- or -

Method 2:

$$\frac{dA}{dt} = 8 - \sqrt{t+1}$$

$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + C$$

$$30 = 8(0) - \frac{2}{3}(0+1)^{3/2} + C; \quad C = \frac{92}{3}$$

$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + \frac{92}{3}$$

- (d) $A'(t) = 8 - \sqrt{t+1} = 0$ when $t = 63$
 $A'(t)$ is positive for $0 < t < 63$ and negative for $63 < t < 120$. Therefore there is a maximum at $t = 63$.

Method 1:

$$3 \begin{cases} 2 : \text{definite integral} \\ 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

- or -

Method 2:

$$3 \begin{cases} 1 : \text{antiderivative with } C \\ 1 : \text{solves for } C \text{ using } L(0) = 0 \\ 1 : \text{answer} \end{cases}$$

1 : answer

Method 1:

$$2 \begin{cases} 1 : 30 + 8t \\ 1 : -\int_0^t \sqrt{x+1} dx \end{cases}$$

- or -

Method 2:

$$2 \begin{cases} 1 : \text{antiderivative with } C \\ 1 : \text{answer} \end{cases}$$

$$3 \begin{cases} 1 : \text{sets } A'(t) = 0 \\ 1 : \text{solves for } t \\ 1 : \text{justification} \end{cases}$$

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Question 2

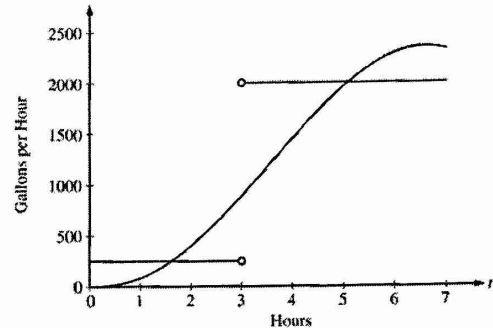
The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \leq t \leq 7$, where t is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is

$$f(t) = 100t^2 \sin(\sqrt{t}) \text{ gallons per hour for } 0 \leq t \leq 7.$$

(ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$



The graphs of f and g , which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

- (a) How many gallons of water enter the tank during the time interval $0 \leq t \leq 7$? Round your answer to the nearest gallon.
- (b) For $0 \leq t \leq 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.
- (c) For $0 \leq t \leq 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

(a) $\int_0^7 f(t) dt \approx 8264$ gallons

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) The amount of water in the tank is decreasing on the intervals $0 \leq t \leq 1.617$ and $3 \leq t \leq 5.076$ because $f(t) < g(t)$ for $0 \leq t < 1.617$ and $3 < t < 5.076$.

2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(c) Since $f(t) - g(t)$ changes sign from positive to negative only at $t = 3$, the candidates for the absolute maximum are at $t = 0, 3$, and 7 .

5 : $\begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{integrand} \\ 1 : \text{amount of water at } t = 3 \\ 1 : \text{amount of water at } t = 7 \\ 1 : \text{conclusion} \end{cases}$

t (hours)	gallons of water
0	5000
3	$5000 + \int_0^3 f(t) dt - 250(3) = 5126.591$
7	$5126.591 + \int_3^7 f(t) dt - 2000(4) = 4513.807$

The amount of water in the tank is greatest at 3 hours. At that time, the amount of water in the tank, rounded to the nearest gallon, is 5127 gallons.