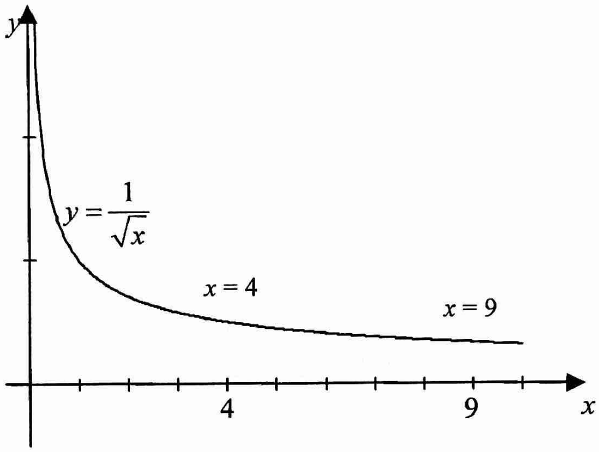
Solutions to Review #7 - Area

1996 AB2
Solution

(a) $\int_4^9 \frac{dx}{\sqrt{x}} dx = 2$



(b) $\int_4^k \frac{dx}{\sqrt{x}} dx = 1$

$2\sqrt{x}\Big]_4^k = 1$

$2\sqrt{k} - 2\sqrt{4} = 1$

$k = \frac{25}{4}$

$\left(\text{or } \int_k^9 \frac{dx}{\sqrt{x}} = 1 \quad \text{or} \quad \int_4^k \frac{dx}{\sqrt{x}} = \int_k^9 \frac{dx}{\sqrt{x}} \right)$

(c) $\text{Volume} = \int_4^9 \left(\frac{1}{\sqrt{x}}\right)^2 dx$

$= \int_4^9 \frac{dx}{x} = \ln x\Big|_4^9 = \ln\frac{9}{4} \quad (\text{or } 0.811)$

1996 BC1
Solution

(a) Volume $= 2\pi \int_0^\infty xe^{-x^2}\,dx$

$$= 2\pi \lim_{b\to\infty} \int_0^b xe^{-x^2}\,dx$$

$$= 2\pi \lim_{b\to\infty}\left[-\frac{1}{2}e^{-x^2}\right]_0^b = 2\pi \lim_{b\to\infty}\left(-\frac{1}{2}e^{-b^2}+\frac{1}{2}e^0\right)$$

$$= 2\pi\left(\frac{1}{2}\right) = \pi$$

or

$$\text{Volume} = \pi \int_0^1\left(\sqrt{(-\ln y)}\right)^2 dy = -\pi \lim_{a\to 0^+}\int_a^1 (\ln y)\,dy = \pi$$

(b) Maximum:

$A(w) = we^{-w^2}$,

$A'(w) = e^{-w^2} - 2w^2e^{-w^2}$

$= e^{-w^2}\left(1-2w^2\right)$.

$A'(w) > 0$ when $w < \frac{1}{\sqrt{2}}$,

$A'(w) = 0$ when $w = \frac{1}{\sqrt{2}}$,

$A'(w) < 0$ when $w > \frac{1}{\sqrt{2}}$.

Therefore, max occurs when $w = \frac{1}{\sqrt{2}}$

Inflection:

$h(x) = e^{-x^2}, h'(x) = -2xe^{-x^2}$,

$h''(x) = -2e^{-x^2} - 2x(-2x)e^{-x^2}$

$= 2e^{-x^2}\left(-1+2x^2\right)$.

$h''(x) < 0$ when $x < \frac{1}{\sqrt{2}}$,

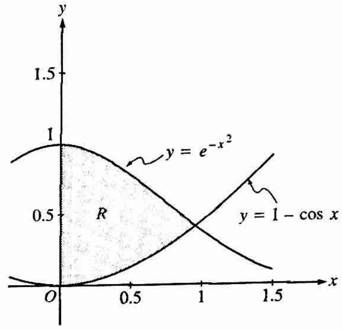
$h''(x) = 0$ when $x = \frac{1}{\sqrt{2}}$,

$h''(x) > 0$ when $x > \frac{1}{\sqrt{2}}$.

Therefore, inflection point when $x = \frac{1}{\sqrt{2}}$.

Therefore, the maximum value of $A(w)$ and the inflection point of $h(x)$ occur when x and w are $\frac{1}{\sqrt{2}}$.

AP Calculus AB–1 / BC–1 **2000**



Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

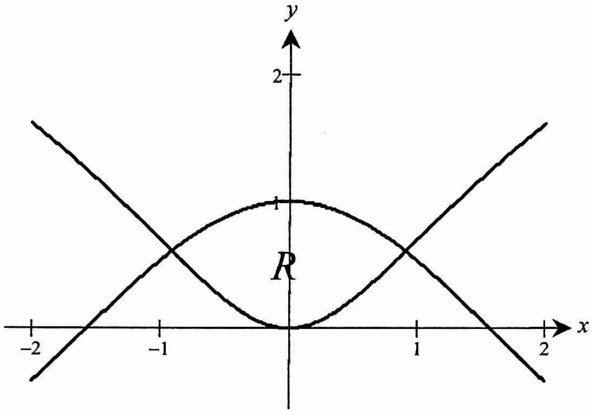
(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

Solution	Scoring
Region R $e^{-x^2} = 1 - \cos x$ at $x = 0.941944 = A$	1 : Correct limits in an integral in (a), (b), or (c).
(a) Area $= \int_0^A \left(e^{-x^2} - (1 - \cos x)\right) dx$ $= 0.590$ or 0.591	2 { 1 : integrand 1 : answer
(b) Volume $= \pi \int_0^A \left(\left(e^{-x^2}\right)^2 - (1 - \cos x)^2\right) dx$ $= 0.55596\pi = 1.746$ or 1.747	3 { 2 : integrand and constant < −1> each error 1 : answer
(c) Volume $= \int_0^A \left(e^{-x^2} - (1 - \cos x)\right)^2 dx$ $= 0.461$	3 { 2 : integrand < −1> each error Note: 0/2 if not of the form $k\int_c^d (f(x) - g(x))^2\, dx$ 1 : answer

1997 BC3
Solution

(a)



$\ln\left(x^2+1\right)=\cos x$

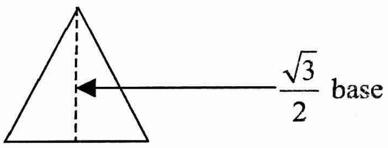
$x=\pm 0.91586$

let $B=0.91586$

$$\text{area} = \int_{-B}^{B}\left[\cos x-\ln\left(x^2+1\right)\right]dx$$
$$=1.168$$

(b) $L=\int_{-B}^{B}\sqrt{1+\left(\frac{2x}{x^2+1}\right)^2}\,dx+\int_{-B}^{B}\sqrt{1+\left(-\sin x\right)^2}\,dx$

(c)

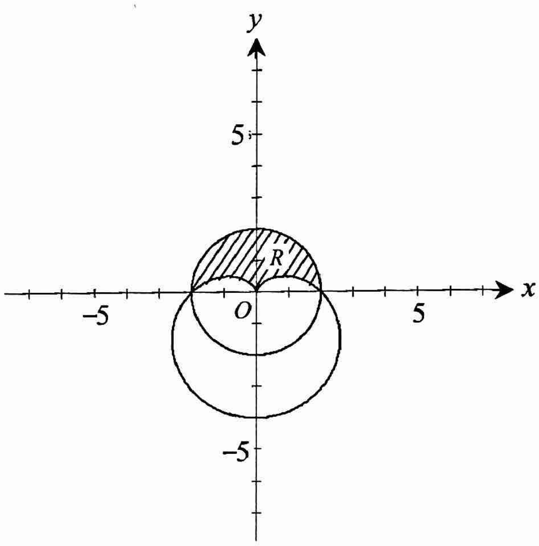


$$\text{area of cross section } =\frac{1}{2}\left[\cos x-\ln\left(x^2+1\right)\right]\times\left[\frac{\sqrt{3}}{2}\left(\cos x-\ln\left(x^2+1\right)\right)\right]$$

$$\text{volume } =\int_{-B}^{B}\frac{\sqrt{3}}{4}\left[\cos x-\ln\left(x^2+1\right)\right]^2 dx$$

1990 BC4
Solution

(a)



(b) $A = \frac{1}{2}\int_0^{\pi}\left[2^2 - \left(2\left(1-\sin\theta\right)\right)^2\right]d\theta$

$= 2\int_0^{\pi}\left(2\sin\theta - \sin^2\theta\right)d\theta$

$= 4\int_0^{\pi}\sin\theta\, d\theta - \int_0^{\pi}\left(1-\cos 2\theta\right)d\theta$

$= -4\cos\theta\Big|_0^{\pi} - \left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\pi}$

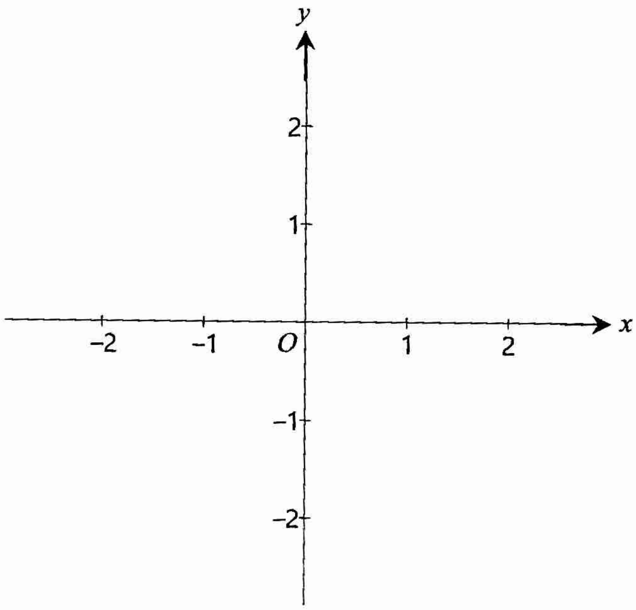
$= \left[-4(-1) + 4(1)\right] - [\pi - 0]$

$= 8 - \pi$

1993 BC4

Consider the polar curve $r = 2\sin(3\theta)$ for $0 \le \theta \le \pi$.

(a) In the *xy*-plane provided below, sketch the curve.



(b) Find the area of the region inside the curve.

(c) Find the slope of the curve at the point where $\theta = \frac{\pi}{4}$.