BC Calculus
Review #6 – Integral Applications

1. 2003B AB3/BC3 (Calculator allowed)

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

A blood vessel is 360 mm long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.

(a) Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.

(b) Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.

(c) Using correct units, explain the meaning of $\pi\int_{125}^{275}\left(\frac{B(x)}{2}\right)^2 dx$ in terms of the blood vessel.

(d) Explain why there must be at least one value x, for $0 < x < 360$, such that $B''(x) = 0$.

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a) radius $= \frac{1}{2}B(x)$

Avg radius $= \frac{1}{360}\int_0^{360} \frac{1}{2}B(x)\,dx$

b) $\frac{1}{360}\int_0^{360}\frac{1}{2}B(x)dx \approx \frac{1}{720}\left[120(30+30+24)\right]$

$\approx \frac{1}{6}(84) = \boxed{14 \text{ cm}}$

c) $\pi\int_{125}^{275}\left(\frac{B(x)}{2}\right)^2 dx$ sum of discs πr^2 with $r = \frac{B(x)}{2}$ and depth dx

This represents the total volume of the blood vessel from 125 mm to 275 mm along the vessel, in mm^3.

d) By the Mean Value Theorem, since $B(60) = 30 = B(180)$, $\exists$ a c_1 on $(60, 180)$ s.t. $B'(c_1) = 0$. Also, since $B(240) = B(360) = 26$, $\exists$ a c_2 on $(240, 360)$ s.t. $B'(c_2) = 0$. Since $B'(c_1) = B'(c_2)$ on $(60, 360)$, $\exists$ a c_3 s.t. $B''(c_3) = 0$ on $(60, 360)$, or more accurately, on (c_1, c_2).

2. 2007 AB5/BC5 (no calculator)

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$.

The radius of the balloon is 30 feet when $t = 5$. (Note: the volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

(a) Estimate the radius of the balloon using the tangent line approximation at $t = 5.4$. Is your estimate greater than or less than the true value? Give a reason for your answer.

(b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

(c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt$ in terms of the radius of the balloon.

(d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\,dt$? Give a reason for your answer.

$r'' < 0$ on $0 < t < 12$, $r(5) = 30$, $V = \frac{4}{3}\pi r^3$

a) $r'(5) = 2 \leftarrow$ Tangent slope @ $t=5$

point @ $t=5$: $(5, 30)$

tangent approx : $y \approx r$ near $t=5$

$y - 30 = 2(t-5)$

$y = 2(t-5) + 30$

For $t = 5.4$

$r \approx y = 2(5.4-5) + 30$

$= 2(0.4) + 30$

$= \boxed{30.8 \text{ ft}}$

Because $r'' < 0$ on $0 < t < 12$, The graph of r is concave down at $t=5$. This means that the tangent line approximation gives an overestimate.

b) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

at $t = 5$

$\frac{dV}{dt} = 4\pi (30)^2 (2)$

$\frac{dV}{dt} = 8\pi \cdot 900 \text{ ft}^3/\text{min}$

c) $\int_0^{12} r'(t)\,dt \approx 2(4) + 3(2) + 2(1.2) + 4(0.6) + 1(0.5)$

this represents the change in the radius, in feet, from $t = 0$ to 12 minutes.

d) $r'' < 0$, so r is concave down and r' is decreasing.

$\therefore$ The righthand Riemann sum is an underestimate (less than $\int_0^{12} r'(t)\,dt$).

3. 2008 AB2/BC2 (calculator allowed)

t (hours)	0	1	3	4	7	8	9
L(t) (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time *t* is modeled by a twice-differentiable function *L* for $0 \le t \le 9$. Values of *L(t)* at various times *t* are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. (*t* = *5.5)*. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

(c) For $0 \le t \le 9$, what is the fewest number of times that $L'(t)$ must equal zero? Give a reason for your answer.

(d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{\frac{-t}{2}}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. (*t* = *3*), to the nearest whole number?

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t=0 on sale. t=9 sold out

a) $L(t)$ = # people in line @ time t

$L'(t)$ = rate of change of # of ppl in line

$$L'(5.5) \approx \frac{L(7)-L(4)}{7-4} = \frac{150-126}{3} = \frac{24}{3} = 8$$

8 people per hour

b) Average $= \frac{1}{4}\int_0^4 L(t)\,dt$

$$\approx \frac{1}{4}\cdot\frac{1}{2}\left[(120+156)1 + (156+176)2 + (176+126)1\right] \text{ people}$$

c) $L(t)$ is twice-differentiable, so it is continuous.

Since $L(3) > L(1)$, $L'(c_1) > 0$ for some $c_1 \in (1,3)$.

Since $L(7) > L(4)$, $L'(c_2) > 0$ for some $c_2 \in (4,7)$

Since $L(4) < L(3)$, $L'(c_3) < 0$ for some $c_3 \in (3,4)$

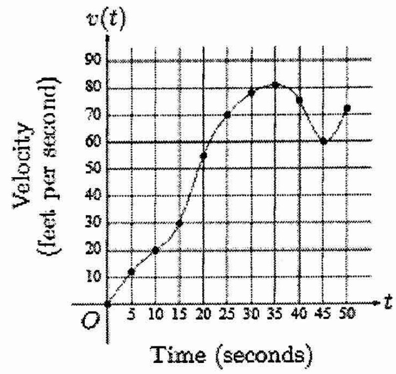
Since $L(8) < L(7)$, $L'(c_4) < 0$ for some $c_4 \in (7,8)$

By the Intermediate Value Theorem, $L'(t) = 0$ for at least three values on $[0,9]$. The derivative changes sign at least 3 times.

d) Total Tickets sold $= \int_0^9 r(t)\,dt = 972.784$

There were approx. 973 tickets sold by 3 pm.

4. 1998 AB3 (calculator allowed)



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

The graph of the velocity *v(t)*, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$ is shown above. A table of values for *v(t)*, at 5 second intervals of time *t*, is shown to the right of the graph.

(a) During what intervals of time is the acceleration positive? Give a reason for your answer.

(b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.

(c) Find an approximation for the acceleration of the car, in ft/sec^2, at *t = 40*. Show the computations you used to arrive at your answer.

(d) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

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a) Because $v(t)$ is increasing on $0<t<35$ and $45<t<50$, acceleration $a(t) = v'(t)$ must be positive on those intervals

b) Average Acceleration $= \frac{1}{50}\int_0^{50} a(t)\,dt = \frac{1}{50}\left[v(50) - v(0)\right] = \frac{1}{50}(72-0)$ ft/sec^2

c) Acceleration at $t=40$ $= a(40) = v'(40) \approx \frac{v(45)-v(35)}{45-35} = \frac{60-81}{10} = \frac{-21}{10}$ ft/sec2

d) $\int_0^{50} v(t)\,dt \approx 10\left[v(5)+v(15)+v(25)+v(35)+v(45)\right]$

$= 10\left[12+30+70+81+60\right]$ gives the total displacement (change in position) of the car, in feet, from $t=0$ sec to $t=50$ sec.