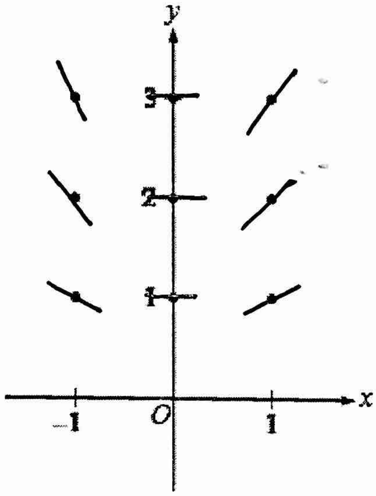
BC Calculus

Review #5 – Slope Fields and Euler's Method

1998 BC4 (calculator allowed)

Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

a) On the axes below, sketch a slope field for the given differential equation at the nine points indicated.



b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 3$. Use Euler's method starting at $x = 0$ with step size of *0.1* to approximate $f(0.2)$. Show the work that leads to your answer.

x	y	Slope	Next y
0	3	0	$3 + 0.1(0) = 3$
0.1	3	0.15	$3 + 0.1(0.15) = 3.015$
0.2	3.015		

$f(0.2) \doteq 3.015$

c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$. Use your solution to find $f(0.2)$.

$$\int \frac{dy}{y} = \int \frac{x}{2}\,dx$$

$$\ln|y| = \frac{x^2}{4} + C, \quad f(0) = 3$$

$$\ln|3| = 0 + C$$

$$\ln 3 = C$$

$$\ln|y| = \frac{x^2}{4} + \ln 3$$

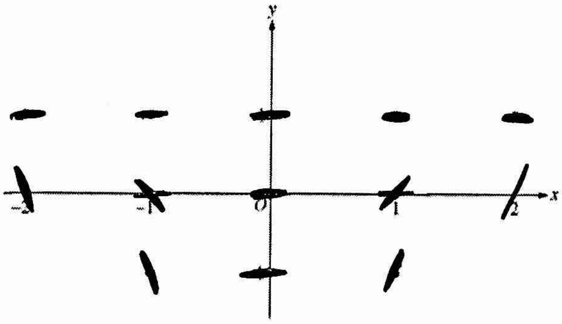
$$y = 3e^{x^2/4}$$

$$y(0.2) = 3e^{(0.2)^2/4} = 3.030$$

2000 BC6 (no calculator)

Consider the differential equation given by $\frac{dy}{dx} = x(y-1)^2$.

a) On the axes provided, sketch a slope field for the given differential equation at the eleven points indicated.



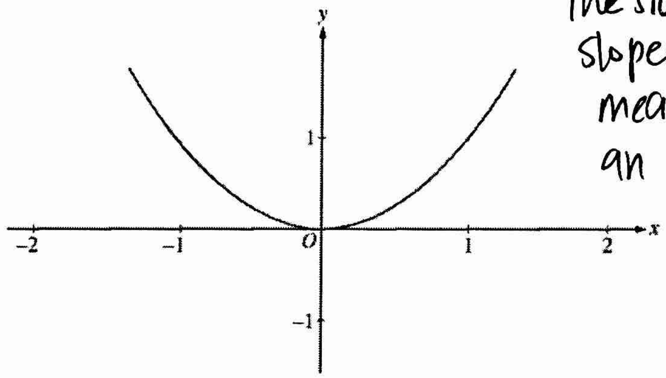
$1(-1)^2 = 1$

$2(-1)^2 = 2$

$1(-2)^2 = 4$

$-1(-2)^2 = -4$

b) Use the slope field for the given differential equation to explain why a solution could not have the graph shown below.



The slope field shows slopes of 0 at $y=1$, meaning that there is an asymptote there

c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -1$.

$$\int \frac{dy}{(y-1)^2} = \int x\,dx$$

$$\int (y-1)^{-2}\,dy = \frac{1}{2}x^2 + C$$

$$-(y-1)^{-1} = \frac{1}{2}x^2 + C$$

$f(0) = -1$

$$\frac{-1}{-1-1} = 0 + C$$

$$\frac{1}{2} = C$$

$$-\frac{1}{y-1} = \frac{1}{2}x^2 + \frac{1}{2}$$

$$-\frac{1}{y-1} = \frac{x^2+1}{2}$$

$$y-1 = \frac{-2}{x^2+1}$$

$$\boxed{y = 1 - \frac{2}{x^2+1}}$$

d) Find the range of the solution found in part c).

$x^2 \geq 0$

$x^2 + 1 \geq 1$

$\frac{2}{x^2+1}$

$0 < \frac{2}{x^2+1} \leq 2$

$0 > \frac{-2}{x^2+1} \geq -2$

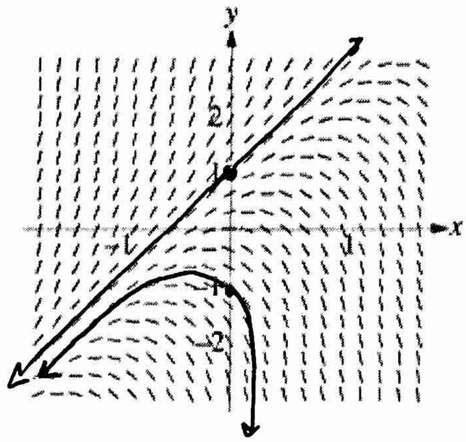
$1 > 1 - \frac{2}{x^2+1} \geq -1$

$\boxed{-1 \leq y < 1}$

2002 BC5 (no calculator)

Consider the differential equation $\frac{dy}{dx} = 2y - 4x$.

a) The slope field for the given differential equation is given. Sketch the solution curve that passes through the point (0, 1) and sketch the solution curve that passes through the point (0, -1).



b) Let f be the function that satisfies the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.

x	y	Slope	Next y
0	1	2	$1+2(0.1) = 1.2$
0.1	1.2	2	$1.2+2(0.1) = 1.4$
0.2	1.4		

$2.4 - 0.4$

$\boxed{f(0.2) \approx 1.4}$

b) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer.

$y = 2x + b$

$\frac{dy}{dx} = 2y - 4x$

$2 = 2(2x+b) - 4x$

$2 = 4x + 2b - 4x$

$2 = 2b$

$\boxed{b = 1}$

d) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of g have a local extremum at the point $(0, 0)$? If so, is the point a local maximum or a local minimum? Justify your answer.

$g'(0) = \left.\frac{dy}{dx}\right|_{\substack{x=0\\y=0}} = 2(0) - 4(0) = 0$

$g''(0) = \left.\frac{d}{dx}\left(\frac{dy}{dx}\right)\right|_{(0,0)} = \left.2y' - 4\right|_{y=0} = 0 - 4 = -4$

Since $g'(0) = 0$ and $g''(0) < 0$, g has a local maximum @ $(0,0)$