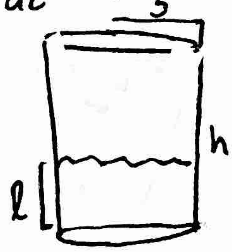
BC Calculus

Review #3 – Derivative Applications

1. Water is flowing into a cylindrical tank of radius 5 meters at the rate of 16 cubic meters per minute. How fast is the water level rising?

$\frac{dV}{dt} = 16$



$V = \pi(25)\ell$

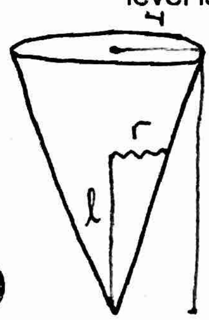
$V = 25\pi\ell$

$\frac{dV}{dt} = 25\pi \frac{d\ell}{dt}$

$16 = 25\pi \frac{d\ell}{dt}$

$\frac{d\ell}{dt} = \boxed{\frac{16}{25\pi} \text{ meters/minute}}$

2. Water is running out of a conical funnel at the rate of 1 cubic in per second. If the radius of the base of the funnel is 4 inches and the height is 8 inches, find the rate at which the water level is dropping when it is 2 inches from the top.



$\frac{dV}{dt} = -1$

when $8 - \ell = 2$

$\ell = 6$

find $\frac{d\ell}{dt}$

$V = \frac{1}{3}\pi r^2 \ell \rightarrow V = \frac{1}{3}\pi\left(\frac{1}{2}\ell\right)^2 \ell$

$\frac{r}{\ell} = \frac{4}{8}$

$\ell = 2r$

$r = \frac{1}{2}\ell$

$V = \frac{\pi}{12}\ell^3$

$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3\ell^2 \frac{d\ell}{dt}$

$-1 = \frac{\pi}{12} \cdot 3(6)^2 \frac{d\ell}{dt}$

$-1 = \frac{54}{\pi}\frac{d\ell}{dt}$

$\frac{d\ell}{dt} = \frac{-\pi}{54}$

The water is dropping @ $\frac{\pi}{54}$ in per second

3. Sand falling from a chute forms a conical pile whose altitude is always equal to 4/3 the radius of the base.

a) How fast is the volume increasing when the radius of the base is 3 feet and is increasing at the rate of 3 inches/minute?

$V = \frac{1}{3}\pi r^2 a$

$a = \frac{4}{3}r$

$r = \frac{3}{4}a$

$V = \frac{1}{3}\pi r^2\left(\frac{4}{3}r\right)$

$V = \frac{4\pi}{9}r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi r^2 \frac{dr}{dt}$

$r = 36, \ \frac{dr}{dt} = 3$

$\frac{dV}{dt} = \frac{4}{3}\pi(36)^2 \cdot 3$

$\frac{dV}{dt} = \boxed{5184\pi \text{ cubic inches/min}}$

b) How fast is the radius increasing when it is 6 feet and the volume is increasing at the rate of 24 cubic feet per minute?

$\frac{dV}{dt} = \frac{4}{3}\pi r^2 \frac{dr}{dt}$

$24 = \frac{4}{3}\pi(6)^2 \frac{dr}{dt}$

$24 = 48\pi \frac{dr}{dt}$

$\frac{1}{2\pi} = \frac{dr}{dt}$

$\boxed{\frac{1}{2\pi} \text{ feet per minute}}$

4. Two parallel sides of a rectangle are being lengthened at the rate of 2 cm/sec, while the other two sides are being shortened in such a way that the figure remains a rectangle with constant area of 50 sq cm. What is the rate of change of the perimeter when the length of an increasing side is

a) 5 cm?

$A = bh = 50$

$\frac{db}{dt} = 2$

$P = 2b + 2h$

$P = 2b + 2\left(\frac{50}{b}\right)$

$p = 2b + 100b^{-1}$

$\frac{dP}{dt} = 2\frac{db}{dt} - 100b^{-2}\frac{db}{dt}$

$\frac{dP}{dt} = 2(2) - \frac{100}{25}(2)$

$= 4 - 4(2)$

$= \boxed{-4 \text{ cm/sec}}$

b) 10 cm?

$\frac{dP}{dt} = 2(2) - 100(100)^{-1}(2)$

$= 4 - 2$

$= \boxed{2 \text{ cm/sec}}$

c) What are the dimensions when the perimeter ceases to decrease?

$\frac{dP}{dt} = 2(2) - \frac{100}{b^2}(2) = 0$

$4 - \frac{200}{b^2} = 0$

$\frac{200}{b^2} = 4$

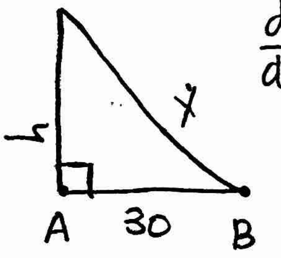
$b^2 = \frac{200}{4}$

$b^2 = 50$

$b = \sqrt{50}$

The dimensions are $\sqrt{50}$ by $\sqrt{50}$

5. A balloon is rising vertically over a point A at the rate of 15 ft/sec. Point B on the ground is level with point A and is 30 ft from point A. At what rate is the distance between B and the balloon changing when the balloon is 40 ft high?



$\frac{dh}{dt} = 15$

$h^2 + 30^2 = x^2$

when $h = 40$,

$x = 50$

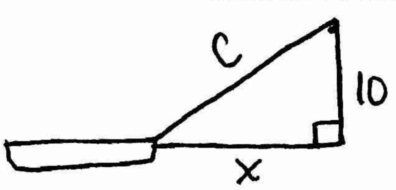
$h^2 + 30^2 = x^2$

$2h \cdot h' + 0 = 2x \cdot x'$

$2(40)(15) = 2(50)x'$

$x' = \boxed{12 \text{ ft/sec}}$

6. A barge whose deck is 10 ft below the level of a dock is being pulled into the dock by means of a cable attached to the deck and passing through a ring on the dock. The barge is approaching the dock at ¾ feet per second. How fast is the cable being pulled in when the boat is 24 feet from the dock?



$\frac{dx}{dt} = \frac{-3}{4}$

when $x = 24$, what is $\frac{dc}{dt}$?

$x^2 + 10^2 = c^2$

$2x \cdot x' = 2c \cdot c'$

$2(24)\left(\frac{-3}{4}\right) = 2(26)c'$

$\frac{24(-3)}{26(4)} = c'$

$\frac{-18}{26} = c'$

$c' = -9/13$

$c^2 = 100 + 24^2$

$c = 26$

The cable is being pulled in at $\frac{-9}{13}$ ft per second