

Final Review #3: Exponential Equations

Percent Increase or Decrease ← always exponential

| Percent Increase: | Percent Decrease: |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| to find <u>growth factor</u> b , $100\% + \underline{\quad}\% = \underline{\quad}\%$ <div style="text-align: right; margin-left: 150px;">↑ b-value in function</div> | to find <u>decay factor</u> b , $100\% - \underline{\quad}\% = \underline{\quad}\%$ <div style="text-align: right; margin-left: 150px;">↑ b-value in function</div> |
| (ex) "increase by 5%" $100\% + 5\% = 105\%$ $= 1.05 \leftarrow b$ | (ex) "decrease by 5%" $100\% - 5\% = 95\%$ $= 0.95 \leftarrow b$ |
| (ex) "increase by 107.2%" $100\% + 107.2\% = 207.2\%$ $= 2.072 \leftarrow b$ | (ex) "decrease by 23.5%" $100\% - 23.5\% = 76.5\%$ $= 0.765 \leftarrow b$ |

Linear vs. Exponential

| Linear: | Exponential: |
|----------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <u>Graph</u> : <u>line</u> | <u>Graph</u> : <u>curve w/ asymptote</u> |
| <u>Equation</u> : $y = mx + b$ <div style="text-align: center; margin-left: 50px;">↑ ↑ slope y-int.</div> | <u>Equation</u> : $y = a \cdot b^x$ <div style="text-align: center; margin-left: 50px;">↑ ↑ y-int growth/decay factor</div> |
| <u>Growth Pattern</u> : "constant rate of change" "increase/decrease by the same <u>amount</u> " | <u>Growth Pattern</u> : "doubling", "tripling" "divide by 2" "multiply by 10" "percent increase" |
| <u>Table</u> : add/subtract same # to y-values each time you increase x. | <u>Table</u> : multiply/divide by the same # to y-values each time you increase x. |

Examples

1. At 2 pm, the population in the sample is 700. It increases by 200 bacteria every hour. How many bacteria will be in the sample at 11 pm? ← $X=9$

LINEAR: $y = mx + b$
 $y = 200x + 700$

$x = \# \text{ hrs after 2 pm}$
 $y = \# \text{ bacteria}$

$y = 200(9) + 700$
 $y = 1800 + 700$
 $y = \boxed{2500 \text{ bacteria}}$

2. At 2 pm, the population in the sample is 1000. It triples every hour. How many bacteria will be in the sample at 5 pm? ← $X=3$

EXPONENTIAL: $y = a \cdot b^x$
 $y = 1000 \cdot 3^x$

$x = \# \text{ hrs since 2 pm}$
 $y = \# \text{ bacteria}$

$y = 1000 \cdot 3^3$
 $y = 1000 \cdot 27$
 $y = \boxed{27000 \text{ bacteria}}$

3. At 2 pm, the population of the sample was 300. The population decreases by 31% each hour. How many bacteria will be in the sample at midnight? ← $X=10$

EXPONENTIAL: $y = a \cdot b^x$

100% - 31%
 = 69%
 = 0.69

$y = 300(0.69)^x$
 $y = 300(0.69)^{10}$
 $y = 7.34$
 $\boxed{7 \text{ bacteria}}$

4. At 2 pm, the population of the sample was 900. The population increases by 7.2% each hour. How many bacteria will be in the sample at 8 pm? ← $X=6$

EXPONENTIAL: $y = a \cdot b^x$

100% + 7.2%
 = 107.2%
 = 1.072

$y = 900(1.072)^x$
 $y = 900(1.072)^6$
 $y = 1365.88$
 $\boxed{1366 \text{ bacteria}}$

Solve by Creating Common Bases

$2^x \cdot 2^{x-5} = 8^{2x+1}$

Common base = 2
 $8 = 2^3$

$2^x \cdot 2^{x-5} = (2^3)^{2x+1}$
 $2^{x+x-5} = 2^{3(2x+1)}$
 $x+x-5 = 3(2x+1)$
 $2x-5 = 6x+3$
 $-2x-3 \quad -2x-3$

$-8 = 4x$
 $\boxed{-2 = x}$

$\left(\frac{1}{3}\right)^x = 3^{x+1} \cdot 9^x$

Common base = 3
 $\frac{1}{3} = 3^{-1} \quad 9 = 3^2$

$(3^{-1})^x = 3^{x+1} \cdot (3^2)^x$
 $3^{-x} = 3^{x+1} \cdot 3^{2x}$
 $3^{-x} = 3^{x+1+2x}$
 $-x = x+1+2x$
 $-x = 3x+1$
 $-3x-3x$
 $-4x = 1$
 $\boxed{x = -\frac{1}{4}}$