

Review #1 - Limits

1) $\lim_{x \rightarrow 2} \frac{x-2}{x-2} = \boxed{1}$

2) $\lim_{x \rightarrow 5} 2x^2 - 4x + 7 = 50 - 20 + 7 = \boxed{37}$

3) $\lim_{x \rightarrow 2} \frac{(x-2)(x+3)}{x-2} = \boxed{5}$

4) $\lim_{x \rightarrow -\infty} \frac{2x+3}{1-x^2} = \boxed{0}$

5) $\lim_{x \rightarrow 9} \frac{x-0}{\sqrt{x}-3}$
 FROM LEFT = $-\infty$
 FROM RIGHT = $+\infty$
 \rightarrow LIMIT D.N.E.

6) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \boxed{1}$

7) $\lim_{x \rightarrow 2} \frac{x}{4-x^2} = \frac{2}{\text{SMALL}}$
 FROM LEFT = $+\infty$
 FROM RIGHT = $-\infty$
 \rightarrow LIMIT DNE

8) $\lim_{x \rightarrow \infty} \frac{x^2+4}{x-x^2} = \boxed{-1}$

9) \boxed{D}

10) $\lim_{x \rightarrow 3^+} \frac{x+3}{x-3} = \frac{6}{\text{small}} = \boxed{+\infty}$

11) $\lim_{x \rightarrow -\infty} \frac{|8x+6|}{4x-2} = \boxed{-2}$

12) $f(x) = \begin{cases} x^2-2, & x < 1 \\ \frac{1}{2}x+1, & x \geq 1 \end{cases}$

a) $\lim_{x \rightarrow 1^+} f(x) = \frac{1}{2} + 1 = \boxed{\frac{3}{2}}$

b) $\lim_{x \rightarrow 1^-} f(x) = 1^2 + (-2) = \boxed{-1}$

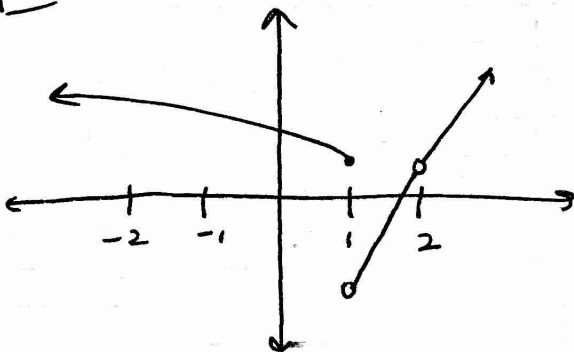
c) $\lim_{x \rightarrow 1} f(x)$ D.N.E. b/c
 $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$

13) $\log \rightarrow \text{power} \rightarrow \text{exponential} \rightarrow \text{factorial}$
 (growth)

14) C

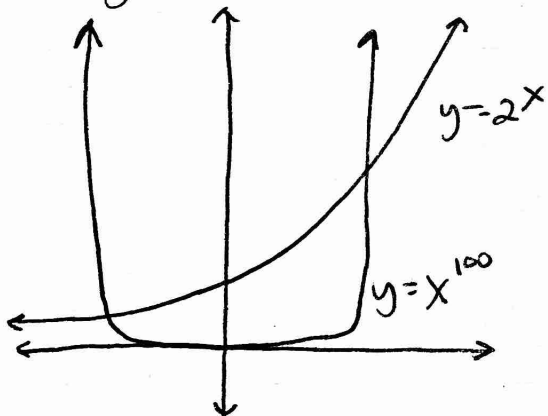
15) E

16)



17) D $\lim_{x \rightarrow -1} f(x)$ D.N.E.

18) They intersect three times, because eventually,



$y = 2^x$ will overtake
 $y = x^{100}$.

19) a) $\lim_{x \rightarrow 1^-} g(x) = 1$ b) $\lim_{x \rightarrow 1^+} g(x) = -2$ c) $\lim_{x \rightarrow -1} g(x) = 1$ d) $g(1) = -2$

20) a) $\lim_{x \rightarrow \infty} \frac{\ln 3x}{x^3} = \boxed{0}$ (order of mag)

b) $\lim_{x \rightarrow \infty} \frac{x^{100}}{e^{0.01x}} = \boxed{0}$ (order of mag)

c) $\lim_{x \rightarrow \infty} \frac{\sqrt{x}}{x} = \boxed{0}$ (order of mag)

21) a) since $\frac{f(x)}{g(x)}$ is increasing, $f(x)$ has the higher order

b) They have the same order

c) g(x) must have a higher order

22) 2^x and x^2 intersect three times

a) Find zeros of $x^2 - 2^x \rightarrow \{-0.767, 2, 4\}$

b) FIRST region: $\int_{-0.767}^2 (2^x - x^2) dx = 2.10592$

SECOND : $\int_2^4 (x^2 - 2^x) dx = 6.6$

TOTAL = 8.773 units

~~22~~ 23) $f(x) = 2xe^{2x}$

a) $\lim_{x \rightarrow -\infty} 2xe^{2x} = -\infty \cdot 0$ indeterminate \rightarrow L'Hospital

$$= \lim_{x \rightarrow -\infty} \frac{2x}{e^{-2x}} = \frac{-\infty}{-\infty}$$

$$= \lim_{x \rightarrow -\infty} \frac{2}{-2e^{-2x}} = \frac{2}{-2(\infty)} = \boxed{0}$$

$$\lim_{x \rightarrow \infty} 2xe^{2x} = \boxed{+\infty}$$

b) Ans. max of $f \rightarrow$ does not exist, because the end behavior is unbounded, and we aren't on a closed interval.

Ans min ..



23 continued

$$f(x) = 2xe^{2x}$$

$$f'(x) = 2e^{2x} + 2x \cdot 2e^{2x}$$

$$f'(x) = 2e^{2x}(1+2x) = 0 \leftarrow \text{crit. value}$$

$$1+2x = 0$$

$$x = -\frac{1}{2}$$

$$f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)e^{-1} = \boxed{-\frac{1}{e}}$$

~~This is~~ This is the absolute minimum because the function has only one critical value on the open interval $(-\infty, \infty)$, and @ this value $x = -\frac{1}{2}$, f' changes from negative to positive.

23c) Range of f : $\boxed{\left(-\frac{1}{e}, \infty\right)}$