AP Calculus BC
Must-Know

Algebra

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Point-slope form: $y - y_0 = m(x - x_0)$

Standard Form: $Ax + By = C$

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Trig Values

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\pi/6$	$1/2$	$\sqrt{3}/2$	$\frac{\sqrt{3}}{3}$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	1	0	∞

Trig Identities

$\sin(2x) = 2\sin x \cos x$

$\cos(2x) = \cos^2 x - \sin^2 x$

$\cos(2x) = 2\cos^2 x - 1$

$\cos(2x) = 1 - 2\sin^2 x$

$\sin^2 x = \frac{1 - \cos 2x}{2}$

$\cos^2 x = \frac{1 + \cos 2x}{2}$

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\sec x = \frac{1}{\cos x}$

$\csc x = \frac{1}{\sin x}$

$\sin(-x) = -\sin(x)$

$\cos(-x) = \cos(x)$

$\tan(-x) = -\tan(x)$

$\cot(-x) = -\cot(x)$

$\sec(-x) = \sec(x)$

$\csc(-x) = -\csc(x)$

Differential Calculus Formulas and Rules

$$\frac{d}{dx}(x)^n = nx^{n-1}$$
$$\frac{d}{dx}(\sin x) = \cos x$$
$$\frac{d}{dx}(\cos x) = -\sin x$$
$$\frac{d}{dx}(\tan x) = \sec^2 x$$
$$\frac{d}{dx}(\cot x) = -\csc^2 x$$
$$\frac{d}{dx}(\sec x) = \sec x \tan x$$
$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(uv) = uv' + vu'$$
$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$
$$\frac{d}{dx}\left(f(g(x))\right) = f'\left(g(x)\right)g'(x)$$
$$\frac{d}{dx}\left(e^x\right) = e^x$$
$$\frac{d}{dx}\left(a^x\right) = a^x \ln a$$
$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$
$$\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$$

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\arccos x) = \frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$
$$\frac{d}{dx}(arc\cot x) = \frac{-1}{1+x^2}$$
$$\frac{d}{dx}(arc\sec x) = \frac{1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}(arccs\mathrm{c}\, x) = \frac{-1}{|x|\sqrt{x^2-1}}$$

Applications of the first and second derivative

Curve Sketching

- To find a critical value, set $f'(x) = 0$ or undefined
- Use a sign chart to determine if the function has a relative extrema. Make sure you write sentences summarizing the results.
- Use can also use the Second Derivative Test to verify extrema. Suppose that x_0 is a critical value. If $f''(x_0) < 0,$ then x_0 is the *x*-coordinate of the relative maximum. If $f''(x_0) > 0,$ then x_0 is the *x*-coordinate of the relative minimum.
- To find points of inflection, set $f''(x) = 0$ or undefined. Then, show that the sign of $f''(x)$ changes as *x* passes through that point.

Three Important Theorems

Intermediate Value Theorem

If a function, $f(x)$ is continuous on a closed interval [a, b] and *y* is some value between $f(a)$ and $f(b),$ then there exists at least one value $x = c$ in the open interval (a, b) where $f(c) = y.$

In other words, a continuous function must pass through every *y*-value between $f(a)$ and $f(b),$.

Mean Value Theorem

If a function, $f(x)$ is continuous on a closed interval [a, b] AND is differentiable on an open interval (a, b), then there exists at least one value $x = c$ in the open interval (a, b) where $f'(c) = \dfrac{f(b) - f(a)}{b - a}.$

In other words, there is at least one point on a smooth curve where the tangent line can be drawn so that it is parallel to the secant line drawn through the endpoints of the interval.

Rolle's Theorem

If a function, $f(x)$ is continuous on a closed interval [a, b] AND is differentiable on an open interval (a, b) AND $f(a) = f(b),$, then there exists at least one value $x = c$ in the open interval (a, b) where $f'(c) = 0.$

In other words, if the endpoints of a differentiable function have the same *y*-coordinates, there is at least one point inside the interval where the slope of the tangent line is equal to zero. This is a special case of the Mean Value Theorem.

Integral Formulas

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c; \; n \neq -1$

$\int \frac{1}{x} \, dx = \ln x + c$

$\int e^x \, dx = e^x + c$

$\int a^x \, dx = \frac{a^x}{\ln a} + c$

$\int \sin x \, dx = -\cos x + c$

$\int \cos \, dx = \sin x + c$

$\int \tan x \, dx = -\ln|\cos x| + c$

$\int \cot \, dx = \ln|\sin x| + c$

$\int \sec x \, dx = \ln|\sec x + \tan x| + c$

$\int \csc x \, dx = -\ln|\csc x + \cot x| + c$

$\int \sec^2 x \, dx = \tan x + c$

$\int \csc^2 x \, dx = -\cot x + c$

$\int \sec x \tan x \, dx = \sec x + c$

$\int \csc x \cot x \, dx = -\csc x + c$

$\int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + c$

$\int \frac{1}{1+x^2} \, dx = \arctan x + c$

$\int \frac{1}{x\sqrt{x^2-1}} \, dx = arc\sec x + c$

$\int \ln x \, dx = x \ln x - x + c$

$\int u \, dv = uv - \int v \, du + c$

(Integration by parts)

Fundamental Theorem of Calculus – Part 1	Fundamental Theorem of Calculus – Part 2	Average Value Theorem
$\int_a^b f'(x) \, dx = f(b) - f(a)$	$\frac{d}{dx}\left(\int_a^x f(t) \, dt\right) = f(x)$	If a function $f(x)$ is continuous on the closed interval [a, b], then there exists some number $x_0 = c$ such that $f(c) = \frac{1}{b-a}\int_a^b f(x) \, dx$
Volume of a Solid of Revolution (disk method) $V = \pi\int_a^b \left((OR)^2 - (IR)^2\right) dx \; or \; dy$	**Volume of a Solid with a Known Cross-Section** $V = \int_a^b Area(x) \, dx$	
	Application Formulas	
$velocity = \frac{d}{dt}(position)$	$acceleration = \frac{d}{dt}(velocity)$	$displacement = \int_{t_1}^{t_2} v(t) \, dt$
$total\ distance = \int_{t_1}^{t_2} \lvert v(t) \rvert \, dt$	$Avg\ velocity = \frac{position_2 - position_1}{time_2 - time_1}$	$Arc\ Length = \int_a^b \sqrt{1 + \left(f'(x)\right)^2} \, dx$
$speed = \lvert velocity \rvert = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$	$velocity\ vector = \left\langle \frac{dt}{dt}, \frac{dy}{dt} \right\rangle$	$acceleration\ vector = \left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right\rangle$

L'Hopital's Rule

If $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$.

Euler's Method

Given that $\frac{dy}{dx} = f(x, y)$ and that the solution passes through the point (x_0, y_0), then $x_{new} = x_{old} + \Delta x$ and $y_{new} = y_{old} + f(x, y) \cdot \Delta x$.

Parametric Functions

Given a function in parametric form $\left(x(t), y(t)\right)$, then $slope = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.

Second derivative in parametric form $\frac{d^2y}{dx^2} = \frac{\frac{d}{d}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$.

Arc length = $\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \; dt$

Polar Functions

$x = r\cos\theta$ and $y = r\sin\theta$.

Slope = $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

Area inside a polar curve = $\frac{1}{2}\int_{\theta_1}^{\theta_2} \left(r(\theta)\right)^2 \; d\theta$ where θ_1 and θ_2 are the first two times that $r = 0$.

Series Stuff

Taylor Series generated by *f(x)* at *x = 0*. (Maclaurin Series)

Let *f* be a function that has derivatives of all order on some open interval containing *x = 0*. Then the Taylor series generated by *f* at *x = 0* is given by

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots + \frac{f^{(n)}(0)}{n!}x^n + \ldots = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!}x^k$$

The partial sum $P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!}x^k$

is the **Taylor polynomial of order *n* for *f* at *x = 0*.**

Taylor Series generated by *f(x)* at *x = a*.

Let *f* be a function that has derivatives of all order on some open interval containing

x = a. Then the Taylor series generated by *f* at *x = a* is given by

$$f(x) = f(a) + f'(a)x + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \ldots$$
$$= \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

The partial sum $P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$

is the **Taylor polynomial of order *n* for *f* at *x = a*.**

Common Maclaurin Series

$$\frac{1}{1-x}=1+x+x^2+x^3+x^4+...+x^n+...=\sum_{n=0}^{\infty}x^n \quad (-1<x<1)$$

$$\frac{1}{1+x}=1-x+x^2-x^3+...+(-x)^{n-1}+...=\sum_{n=0}^{\infty}(-1)^n x^n \quad (-1<x<1)$$

$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}+...+(-1)^{n-1}\frac{x^n}{n}+...=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n} \quad (-1<x\le 1)$$

$$\arctan x=x-\frac{x^3}{3}+\frac{x^5}{5}+...+\left(-1\right)^n\frac{(x)^{2n+1}}{2n+1}+...=\sum_{n=0}^{\infty}\left(-1\right)^n\frac{(x)^{2n+1}}{2n+1} \quad (-1\le x\le 1)$$

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+...+\frac{x^n}{n!}+...=\sum_{n=0}^{\infty}\frac{x^n}{n!} \quad (\text{all real } x)$$

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}+...+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+...=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} \quad (\text{all real } x)$$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}+...+(-1)^n\frac{x^{2n}}{(2n)!}+...=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} \quad (\text{all real } x)$$

Error Bound

Alternating Series Error Bound

When a series is alternating, the error is maximized in the next unused term evaluated at the difference between the center of the interval of convergence and the *x*-coordinate being evaluated.

Lagrange Error Bound

When the Taylor series doesn't alternate, we still find the error by using the Lagrange error bound. The error is still tied to the next unused term according to $Error<\left|\frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}\right|$ where $f^{(n+1)}(z)$ is the maximum value that the corresponding derivative takes on the given interval and *x* is the value of the polynomial function centered at *a*.