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12.2 – Chapter 6 Test Review

Topics requested by classmates – see your full list of topics, handed out last block day

Riemann Sums

Purpose: Approximate the area under a curve.

Tips/Method:

1. Draw the x-axis as a partitioned number line on [a ,b]
2. Divide into *n* subintervals
3. Write an expression for the k[th] rectangle/trapezoid area as a function of k.

Use a calculator to evaluate the left, right, midpoint, and trapezoidal sums with 20 regular subintervals for the function $y = \cos x + 3x$ on the interval [0, 5]. $\Delta X = \frac{5}{n} = \frac{5}{20} = \frac{1}{4}$

$0, \frac{5}{n}, 2(\frac{5}{n}), 3(\frac{5}{n}), \ldots, 20(\frac{5}{n})$

LEFT:
$$\sum_{k=1}^{20} \left(\frac{1}{4}\right)\left[\cos\left((k-1)\frac{1}{4}\right) + \frac{3}{4}(k-1)\right] \doteq 34.761$$

MID:
$$\sum_{k=1}^{20} \left(\frac{1}{4}\right)\left[\cos\left(\frac{k}{4}-\frac{1}{8}\right) + 3\left(\frac{k}{4}-\frac{1}{8}\right)\right] = 36.539$$

RIGHT:
$$\sum_{k=1}^{20} \left(\frac{1}{4}\right)\left[\cos\left(\frac{k}{4}\right) + \frac{3k}{4}\right] \doteq 38.332$$

TRAP:
$$\sum_{k=1}^{20} \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\cos\frac{k}{4} + \frac{3k}{4} + \cos\left(\frac{k-1}{4}\right) + \frac{3(k-1)}{4}\right) = 36.546$$

Integrals at discontinuities/absolute value integrals

Remember:
A function is integrable on an interval if it has a finite number of discontinuities and is bounded on that interval.

Tips/Method for absolute value:

1. Find the roots of the function (if graphing calculator is available, you can use it to do this).
2. Use the roots to find intervals on which the function is negative
3. For those intervals, multiply the function by -1 to make it positive
4. Integrate over adjacent subintervals.

$\int_{-5}^{6} |x^3 - 9x|\, dx$

Analyze $f(x) = x^3 - 9x$:

roots:
$0 = x(x+3)(x-3)$
$x = \{0, \pm 3\}$

where is it negative:

$-3 \quad 0 \quad 3$

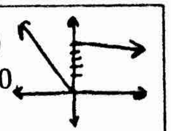
$f(x)\quad - \quad + \quad - \quad +$

$$|x^3 - 9x| = \begin{cases} x^3 - 9x, & -3 < x < 0 \text{ or } x > 3 \\ -x^3 + 9x, & x \le -3 \text{ or } 0 \le x \le 3 \end{cases}$$

Integral =
$$\int_{-5}^{-3} (-x^3 + 9x)\,dx + \int_{-3}^{0} (x^3 - 9x)\,dx + \int_{0}^{3} (-x^3 + 9x)\,dx + \int_{3}^{6} (x^3 - 9x)\,dx$$

CALCULATOR
$$= \boxed{\frac{1147}{4}}$$

$$\int_{-5}^{6} f(x)dx \text{ if } f(x) = \begin{cases} 5, x > 0 \\ -x, x \le 0 \end{cases}$$



$$= \int_{-5}^{0} -x\,dx + \int_{0}^{6} 5\,dx$$
$$= \left.\frac{-x^2}{2}\right|_{-5}^{0} + \left.5x\right|_{0}^{6}$$
$$= 0 - \left(-\frac{(25)}{2}\right) + 30 - 0$$
$$= \boxed{\frac{85}{2}}$$

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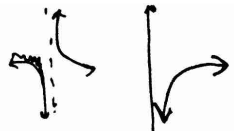
$\frac{1}{x}$  $\ln|x|$

Definite Integrals by u-substitution

Purpose:
Integrate a function that is not a "simple" integral or sum/difference of simple integrals.

Method 1: Find the indefinite integral first (in terms of the original variable), then evaluate the definite integral.

Method 2: Change the limits of integration to u-values.

$$\int_{-2}^{0} 2t^2\sqrt{1-4t^3}\,dt \qquad u = 1-4t^3 \quad du = -12t^2dt \quad -\tfrac{1}{6}du = 2t^2dt$$

Change limits:

$$\int_{-2}^{0} \quad \leftarrow t=0,\ u=1 \qquad \leftarrow t=-2,\ u=1-4(-8)=33$$

$$-\frac{1}{6}\int_{33}^{1} u^{1/2}du = \left(-\frac{1}{6}\right)\left(\frac{2}{3}u^{\frac{3}{2}}\right)\Big|_{33}^{1}$$

$$= -\frac{1}{6}\left(\frac{2}{3} - \frac{2}{3}(33)^{3/2}\right) = \boxed{\frac{1}{9}\left(33\sqrt{33}-1\right)} \doteq 20.952$$

$$\int_{-2}^{-6} \frac{4}{(1+2x)^3} - \frac{5}{1+2x}\,dx \qquad u = 1+2x \quad \tfrac{1}{2}du = dx \qquad x=-6 \to u=-11 \qquad x=-2 \to u=-3$$

$$\frac{1}{2}\int_{-3}^{-11}\left(4u^{-3} - 5u^{-1}\right)du$$

$$= \frac{1}{2}\left(-2u^{-2} - 5\ln|u|\right)\Big|_{-3}^{-11}$$

$$= \frac{1}{2}\left[\frac{-2}{121} - 5\ln|-11|\right] - \frac{1}{2}\left[\frac{-2}{9} - 5\ln 3\right]$$

$$\doteq -3.145$$

Mean Value Theorem for Integrals and Average Value, and Motion Applications

A particle's acceleration is $a(t) = \sin(2t+1)$. It starts at rest, 2 feet from the origin.

A) What is its average velocity on the interval [5, 10]?

$$V_{ave} = \frac{1}{10-5}\int_5^{10} v(t)\,dt$$

Find $v(t)$:

$$v(t) = \int a(t)\,dt = \int \sin(2t+1)\,dt \qquad u = 2t+1 \quad \tfrac{du}{2} = dt$$

$$= \frac{1}{2}\int \sin u\, du = \frac{1}{2}(-\cos u) + C$$

$$v(t) = -\frac{1}{2}\cos(2t+1) + C$$

$v(0) = 0$

$$0 = -\tfrac{1}{2}\cos(1) + C$$

$$C = \tfrac{1}{2}\cos 1$$

$$v(t) = -\tfrac{1}{2}\cos(2t+1) + \tfrac{1}{2}\cos 1$$

Vave

$$V_{ave} = \frac{1}{5}\left(\frac{1}{2}\right)\int_5^{10}\left(-\cos(2t+1) + \cos 1\right)dt \qquad u = 2t+1 \quad \tfrac{1}{2}du = dt$$

$$= \frac{1}{10}\int_5^{10} \frac{1}{2}\cdot(-\cos u)\,du + \cos 1 \cdot t\Big|_5^{10}$$

... calculator ...

$= 0.178$ ft per (unit of time)

B) Is there a time when it has this exact velocity? How do you know? At what time(s)? *Where* is it at those times?

yes, because according to the MVT for integrals, if $v(t)$ is a continuous function, then there exists a value for t_* such that

$$(10-5)\,v(t_*) = \int_5^{10} v(t)\,dt.$$

Solve for t_*:

$$v(t_*) = 0.178$$

$$-\tfrac{1}{2}\cos(2t_*+1) + \tfrac{1}{2}\cos 1 = 0.178$$

$$-\tfrac{1}{2}\cos(2t_*+1) = -0.092$$

$$\cos(2t_*+1) = 0.184$$

$$2t+1 = \pm\arccos(0.184) + 2\pi k$$

$$\boxed{t = \frac{-1 \pm \arccos(0.184) + 2\pi k}{2}}$$

input this (obviously way too involved for exam question).

WHERE IS IT:

$$p(t) = \int v(t)\,dt$$

$$p(t) = -\tfrac{1}{4}\sin(2x+1) + \tfrac{\cos 1}{2}x + C$$

$$2 = -\tfrac{1}{4}\sin(1) + C$$

$$C = -2 + \tfrac{\sin 1}{4}$$

$$p(t) = \frac{-\sin(2x+1)}{4} + \frac{(\cos 1)x}{2} - 2 + \dots$$

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Functions Defined by Integrals, Chain Rule, etc.

$$f(x) = 2x + 1, \qquad g(x) = 6 + \int_5^x f(t)\,dt$$

A) What is $g(3)$? $g(13)$?

$g(3) = 6 + \int_5^3 (2t+1)\,dt$

$= 6 + (t^2+t)\Big|_5^3$

$= 6 + (9+3) - (25+5)$

$= 6 + 12 - 30$

$g(3) = -12$

$g(13) = 6 + \int_5^{13} (2t+1)\,dt$

$= 6 + (t^2+t)\Big|_5^{13}$

$= 6 + (169+13) - (30)$

$\boxed{g(13) = 158}$

B) Locate the relative extrema of g and justify your answers.

$g'(x) = 0 + 2x + 1 \leftarrow$ FTC pt II

$g'(x) = 0$ when $x = -\frac{1}{2}$

g': $-$ | $-\frac{1}{2}$ | $+$

g has a relative minimum at $x = -\frac{1}{2}$ because g' changes from negative to positive around $x = -\frac{1}{2}$

C) Write in closed form: $\frac{d}{dx}\left[\int_5^{f(x)} 3t^2\,dt\right]$

$= 3[f(x)]^2 \cdot f'(x)$

$= 3(2x+1)^2 \cdot 2$

$= 6(2x+1)^2$

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More applications (including review: related rates and optimization)

Suppose that a country has coal reserves of 50 million tons. Based on population projections, the rate of consumption, R, in millions of tons, is expected to increase according to the formula $R = 6.5e^{0.02t}$, where t is in years. If the country uses only its own reserves, estimate how many years the reserves will last.

$\int_0^x 6.5e^{0.02t}dt = 50$

$u = 0.02t \quad du = 0.02dt$

$50du = dt$

$50 \cdot 6.5 \int_0^{0.02x} e^u du = 50$

$325 e^u \Big|_0^{0.02x} = 50$

$325(e^{0.02x} - e^0) = 50$

$e^{0.02x} - 1 = \frac{50}{325}$

$e^{0.02x} = 1 + \frac{50}{325}$

$0.02x \doteq \ln(1.154)$

$x \doteq \frac{\ln 1.154}{0.02}$

$\doteq 7.155$ years

When a person takes a 100 mg tablet of an asthma drug orally, the rate R at which the drug enters the bloodstream is predicted to be $R = 5(0.95^t)$. If the blood does not contain any trace of the drug when the tablet is taken, determine the time in minutes needed for 50 mg to enter the bloodstream.

Total amount in bloodstream

$= \int_0^x 5(0.95^t)dt$

$= 5 \cdot \frac{0.95^t}{\ln(0.95)}\Big|_0^x$

$= \frac{5 \cdot 0.95^x}{\ln(0.95)} - 5 \cdot \frac{1}{\ln(0.95)}$

$\frac{5}{\ln(.95)}\left[0.95^x - 1\right] = 50$

$0.95^x - 1 = 10\ln(0.95)$

$0.95^x = 10\ln(0.95) + 1$

$x = \log_{0.95}\left[10\ln .95 + 1\right]$

$\doteq 14.024$ minutes

What is the x-coordinate of the point on the line y=2x-3 that is closest to the origin

Minimize distance

$d(x) = \sqrt{(x-0)^2 + (2x-3-0)^2}$

$= \sqrt{x^2 + 4x^2 - 12x + 9}$

$= \sqrt{5x^2 - 12x + 9}$ ← MIN!

$d'(x) = \frac{10x - 12}{2\sqrt{5x^2 - 12x + 9}}$

$d'(x) = 0$ when

$10x = 12$

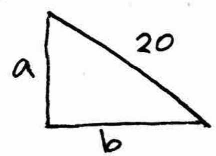
$x = 6/5$

(doesn't make denominator 0 or undefined).

$d' \quad - \quad \frac{6}{5} \quad +$

The line is closest to the origin at $x = \frac{6}{5}$

A ladder 20 feet long leans against a building. If the bottom of the ladder slides away from the building horizontally at a rate of 4 ft/sec, how fast is the ladder sliding down the house when the top of the ladder is 8 feet from the ground.



$a^2 + b^2 = 20^2 \qquad \frac{db}{dt} = 4 \qquad$ find $\frac{da}{dt}$ @ $a = 8$.

$2a \cdot \frac{da}{dt} + 2b \cdot \frac{db}{dt} = 0$

$2(8)\frac{da}{dt} + 2\sqrt{336}(4) = 0$

$16\frac{da}{dt} = -8\sqrt{336}$

$\frac{da}{dt} = \frac{-\sqrt{336}}{2}$

$8^2 + b^2 = 20^2$

$b = \sqrt{400-64}$

$b = \sqrt{336}$

It is sliding down at $\frac{\sqrt{336}}{2}$ ft/sec