8.1 HW

1) $f(x) = \frac{1}{1-2x}$

a) $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots + x^n + \ldots$

$f(x) = \frac{1}{1-2x} = 1 + 2x + (2x)^2 + (2x)^3 + \ldots + (2x)^n + \ldots$

b) $\lim_{n\to\infty} \left| \frac{(2x)^{n+1}}{(2x)^n} \right| = \lim_{n\to\infty} |2x| = |2x|$

converges if $|2x| < 1$

$-1 < 2x < 1$

$-1/2 < x < 1/2$

Check $x = -1/2$

$\sum_{n=0}^{\infty} [2(-1/2)]^n$

$\lim_{n\to\infty} (2(-1/2))^n$ D.N.E

$\Rightarrow$ Diverges by n^{th} term test

Check $x = 1/2$

$\sum_{n=0}^{\infty} [2(1/2)]^n$

$\lim_{n\to\infty} [2(1/2)]^n = 1 \neq 0$

$\Rightarrow$ Diverges by n^{th} term test

$\boxed{(-1/2, 1/2)}$

c) $f(-1/4) \approx 1 + 2(-1/4) + 4(-1/4)^2 + 8(-1/4)^3$

$f(-1/4) \approx \frac{1}{2} + \frac{-4}{16} + \frac{-8}{64}$

$f(-1/4) \approx \frac{4}{8} - \frac{2}{8} - \frac{1}{8}$

$\boxed{f(-1/4) \approx \frac{1}{8}}$

2) Radius for $\sum_{n=0}^{\infty} \frac{(2x-5)^n}{n!}$

$$\lim_{n\to\infty} \left| \frac{(2x-5)^{n+1}}{(n+1)!} \cdot \frac{n!}{(2x-5)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{2x-5}{n+1} \right| \qquad \text{radius} = \infty$$

$$= |0| < 1 \text{ always}$$

3) Interval for $\sum_{n=0}^{\infty} \frac{(x^3-2)^{2n}}{4^n}$

$$\lim_{n\to\infty} \left| \frac{(x^3-2)^{2n+2}}{4^{n+1}} \cdot \frac{4^n}{(x^3-2)^{2n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(x^3-2)^2}{4} \right| = \left| \frac{(x^3-2)^2}{4} \right|$$

converges if $\left| \frac{(x^3-2)^2}{4} \right| < 1$

$$0 < (x^3-2)^2 < 4$$

$$-2 < x^3-2 < 2$$

$$0 < x^3 < 4$$

$$\boxed{0 < x < \sqrt[3]{4}}$$

Check $x = 0$

$$\sum_{n=0}^{\infty} \frac{(-2)^{2n}}{4^n}$$

$$= \sum_{n=0}^{\infty} 1 \qquad \lim_{n\to\infty} 1 = 1$$

Div. by the n^{th} term test

Check $x = \sqrt[3]{4}$

$$\sum_{n=0}^{\infty} \frac{(4-2)^{2n}}{4^n}$$

$$= \sum_{n=0}^{\infty} 1 \quad \text{same as above}$$

4) let $\sum a_k = \sum_{n=1}^{\infty} \frac{1}{\sqrt{3n+5}}$ $\quad \sum_{n=1} b_k = \sum_{n=1}^{\infty} \frac{1}{\sqrt{3n^2}}$

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{3n^2}} = \frac{1}{\sqrt{3}} \sum_{n=1}^{\infty} \frac{1}{n}$$ which diverges by p-series.

$$\frac{1}{\sqrt{3n^2}} < \frac{1}{\sqrt{3n+5}}$$ $\therefore \sum a_k$ also diverges by D.C.T