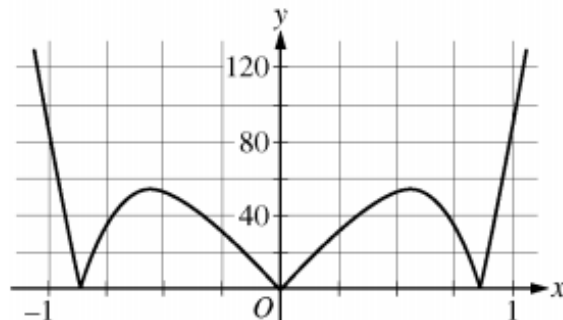


No calculator allowed

2011 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS



Graph of  $y = |f^{(5)}(x)|$

6. Let  $f(x) = \sin(x^2) + \cos x$ . The graph of  $y = |f^{(5)}(x)|$  is shown above.
- (a) Write the first four nonzero terms of the Taylor series for  $\sin x$  about  $x = 0$ , and write the first four nonzero terms of the Taylor series for  $\sin(x^2)$  about  $x = 0$ .
- (b) Write the first four nonzero terms of the Taylor series for  $\cos x$  about  $x = 0$ . Use this series and the series for  $\sin(x^2)$ , found in part (a), to write the first four nonzero terms of the Taylor series for  $f$  about  $x = 0$ .
- (c) Find the value of  $f^{(6)}(0)$ .
- (d) Let  $P_4(x)$  be the fourth-degree Taylor polynomial for  $f$  about  $x = 0$ . Using information from the graph of  $y = |f^{(5)}(x)|$  shown above, show that  $\left| P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right) \right| < \frac{1}{3000}$ .