AP Calculus BC Name:

4'.2 Do Now – Test Review

1. The rate of change of the population of dogs in a dog ranch is jointly proportional to the population and the difference between the population and the carrying capacity. The dog ranch can sustain 100 dogs. It begins with just two dogs, and the population would grow at 1.2% per year if uninhibited.
   a. Where are the asymptotes of the solutions to the differential equation that models this growth pattern?
   b. When will the population reach 50 dogs?

$$\frac{dy}{dt} = \frac{0.012}{100}(y)(100-y)$$

rate of change! proportional! Dog pop! Max!

ⓐ Asymptotes @ min & max
$y=0$ $y=100$

ⓑ $y=50$, find t.

$$\int \frac{dy}{y(100-y)} = \int 0.00012\,dt$$

$$\frac{1}{100}\int\frac{dy}{y} - \frac{1}{100}\int\frac{dy}{100-y} = \int 0.00012\,dt$$

$$\ln\left(\frac{y}{100-y}\right) = 0.012t + C$$

General soln!

b cont... $y(0)=2$

$$\Rightarrow \ln\left(\frac{2}{100-2}\right) = 0.012(0)+C$$

$$C = \ln\left(\frac{2}{98}\right)$$

$$\ln\left(\frac{y}{100-y}\right) = 0.012t + \ln\left(\frac{1}{49}\right)$$

particular soln!

make $y=50$...

$$\ln\left(\frac{50}{100-50}\right) = 0.012t + \ln\left(\frac{1}{49}\right)$$

$$0.012t = -\ln\left(\frac{1}{49}\right)$$

$$t = \frac{\ln 49}{0.012} \approx 324.318 \text{ years} \ldots$$

2. Solve the initial value problem and give the domain of the solution

$y' = e^{-y}(2x-4) \qquad y(5)=0$

$$\frac{dy}{dx} = \frac{2x-4}{e^y}$$

$$\int e^y\,dy = \int 2x-4\,dx$$

$$e^y = x^2-4x+C, \quad y(5)=0$$

← general soln

$$e^0 = 5^2-4(5)+C$$

$$1 = 5+C$$

$$-4 = C$$

$$e^y = x^2-4x-4 \leftarrow \text{implicit soln}$$

$$\boxed{y = \ln(x^2-4x-4)} \leftarrow \text{explicit soln.}$$

Domain: $\frac{dy}{dx} = \frac{2x-4}{e^y} \leftarrow e^y \neq 0$ but that's impossible anyway.

$$e^y = x^2-4x-4$$

positive must be positive

$$x^2-4x-4 \geq 0$$

Find roots...

$$x = \frac{4\pm\sqrt{16-4(-4)}}{2} = \frac{4\pm4\sqrt{2}}{2} = 2\pm2\sqrt{2}$$

$2-2\sqrt{2}$ $2+2\sqrt{2}$

positive on $(-\infty, 2-2\sqrt{2})$ $(2+2\sqrt{2}, \infty)$

contains initial value

Check $y=\ln(x^2-4x-4)$
same restriction. $x^2-4x-4 \geq 0$

$$\boxed{(2+2\sqrt{2}, \infty)}$$

AP Calculus BC

Name:

3. Does the series converge? If so, find its sum

$$\sum_{n=1}^{\infty}\left[\frac{1}{n+2}-\frac{1}{n}\right]=$$

$$S_n=\left[\frac{1}{3}-\frac{1}{1}\right]+\left[\frac{1}{4}-\frac{1}{2}\right]+\left[\frac{1}{5}-\frac{1}{3}\right]+\left[\frac{1}{6}-\frac{1}{4}\right]+\left[\frac{1}{7}-\frac{1}{5}\right]$$

$$+\left[\frac{1}{8}-\frac{1}{6}\right]+\left[\frac{1}{9}-\frac{1}{7}\right]+\ldots\ldots+\underbrace{\left[\frac{1}{n+1}-\cancel{\frac{1}{n-1}}\right]}_{a_{n-1}}+\underbrace{\left[\frac{1}{n+2}-\cancel{\frac{1}{n}}\right]}_{a_n}$$

I notice that $\frac{-1}{1}$ and $-\frac{1}{2}$ "stay", and also the 1st term in the last two binomials

$$S_n=-1-\frac{1}{2}+\frac{1}{n+1}+\frac{1}{n+2} \leftarrow n^{th} \text{ partial sum}$$

$$\lim_{n\to\infty} S_n=-1-\frac{1}{2}+0+0=\boxed{-\frac{3}{2}}$$

← n^{th} partial sum has a finite limit as $n\to\infty$, so the series converges to this limit.

4. Use an appropriate 2nd-degree Taylor polynomial to approximate sin 1, then find the Lagrange error bound for your approximation

$P_2(x)$ use $x_0=\frac{\pi}{3}$, $x=1$, $f(x)=\sin x$

$$P_2(x)=\frac{\sin(\frac{\pi}{3})}{0!}+\frac{\cos(\frac{\pi}{3})(x-\frac{\pi}{3})}{1!}+\frac{-\sin(\frac{\pi}{3})(x-\frac{\pi}{3})^2}{2!}$$

$$P_2(1)=\frac{\sqrt{3}}{2}+\frac{1}{2}\left(1-\frac{\pi}{3}\right)-\frac{\sqrt{3}}{2}\left(1-\frac{\pi}{3}\right)^2\cdot\frac{1}{2}\approx 0.841 \quad \therefore \boxed{\sin 1\approx 0.841}$$

ERROR

$$|R_2(x)|\le\left|\frac{M(x-\frac{\pi}{3})^3}{3!}\right|$$

$x=1$

Find M:

$f^{(3)}(x)=-\cos x$

$f'''(\frac{\pi}{3})=-\frac{1}{2}$

$f'''(1)=-0.5403$

$$|R_2(1)|\le\left|\frac{-0.5403(1-\frac{\pi}{3})^3}{3!}\right|$$

$\le 0.0000094677\ldots$

i.e. it's a good estimate