AP CALCULUS BC

16.2 – More Integration Practice

1. $\int x^2\sqrt{4x^3-5}\,dx =$

$u = 4x^3-5 \quad du = 12x^2\,dx$

$= \frac{1}{12}\int u^{1/2}\,du$

$= \frac{1}{12}\cdot\frac{2}{3}u^{3/2} + C$

$= \boxed{\frac{1}{18}(4x^3-5)^{3/2} + C}$

2. $\int \frac{(\ln x)^3}{x}\,dx =$

$u = \ln x \quad du = \frac{1}{x}\,dx$

$= \int u^3\,du$

$= \frac{1}{4}u^4 + C$

$= \boxed{\frac{1}{4}(\ln x)^4 + C}$

3. $\int \frac{e^x}{e^x+1}\,dx =$

$u = e^x+1, \quad du = e^x\,dx$

$= \int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \boxed{\ln(e^x+1) + C}$

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4. $\int \frac{3}{x^2+25}\,dx =$

$= 3\int \frac{1}{x^2+25}\,dx$

$= 3\int \frac{1}{25\left(\frac{x^2}{25}+1\right)}\,dx$

$= \frac{3}{25}\int \frac{1}{\left(\frac{x}{5}\right)^2+1}\,dx$ ← arctan!

$u = \frac{x}{5} \quad du = \frac{1}{5}\,dx$

$= \frac{3}{25}\cdot 5 \cdot \int \frac{1}{u^2+1}\,dx$

$= \frac{3}{5}\arctan u + C$

$\frac{3}{5}\arctan\left(\frac{x}{5}\right) + C$

5. $\int \frac{x^3-2x^2-4x-5}{x+1}\,dx$ ← long division

$$\begin{array}{r} x^2 - 3x - 1 - \frac{4}{x+1} \\ x+1\overline{)\,x^3-2x^2-4x-5} \\ -(x^3+x^2) \\ \hline -3x^2-4x \\ -(-3x^2-3x) \\ \hline -x-5 \\ -(-x-1) \\ \hline -4 \end{array}$$

$= \int x^2\,dx - \int 3x\,dx - \int dx - \int \frac{4}{x+1}\,dx$ ← Note: could do u-sub & factor out 4

$= \frac{x^3}{3} - \frac{3}{2}x^2 - x - 4\ln|x+1| + C$

6. $\int \cos^3 5x\,dx$

$= \int \cos^2(5x)\cos(5x)\,dx$

$u = 5x \quad du = 5\,dx$

$= \frac{1}{5}\int \cos^2 u \cos u\,du$

$= \frac{1}{5}\int (1-\sin^2 u)\cos u\,du$

$= \int \frac{1}{5}\,du - \frac{1}{5}\int \sin^2 u \cos u\,du$

$= \frac{1}{5}u - \frac{1}{5}$