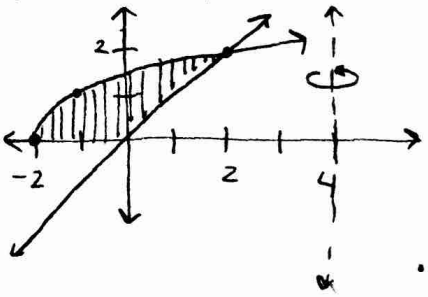
AP CALCULUS BC

16.1 DN – Review for Test

1. *Calculator allowed (but not necessary):* Find the volume of the solid generated when the region bounded by $g(x) = \sqrt{x+2}$, $f(x) = x$, and x-axis is rotated about the line $x = 4$.



$y = \sqrt{x+2}$

$x = y^2 - 2$

$x = y$

dy

$\pi \int_0^2 [(4-y^2+2)^2 - (4-y^2)^2]\, dy$

≈ 87.127

2. *Calculator allowed*: Find the volume and surface area of the solid of revolution generated when the curve $y = x \sin x$ on $[0, \pi]$ is rotated around the x-axis.

Volume

$= \pi \int_0^{\pi} x^2 \sin^2 x\, dx$

$\doteq 13.767$

S.A $= 2\pi \int_a^b f(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$= 2\pi \int_0^{\pi} x \sin x \sqrt{1 + (\sin x + x\cos x)^2}\, dx$

≈ 29.965

3. $\int \tan^{-1}(x-1)\, dx =$

$y = x - 1 \quad dy = dx$

$= \int \tan^{-1} y\, dy$

$u = \tan^{-1} y \quad du = \frac{1}{y^2+1} dy$

$v = y \quad dv = dy$

$= y \tan^{-1} y - \int \frac{y}{y^2+1} dy$

$u = y^2 + 1 \quad \frac{1}{2} du = dy$

$= y \tan^{-1} y - \int \frac{1}{2} \frac{1}{u} du$

$= y \tan^{-1} y - \frac{1}{2} \ln|u| + C$

$= y \tan^{-1} y - \frac{1}{2} \ln|y^2+1| + C$

$(x-1)\tan^{-1}(x-1) - \frac{1}{2} \ln((x-1)^2+1) + C$

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4. $\int_2^\infty \frac{9}{(1-3x)^4}\,dx$

$= \lim_{b\to\infty} \int_2^b \frac{9}{(1-3x)^4}$ $\quad \int_2^b \to \int_{-5}^{1-3b}$

$u = 1-3x \quad du = -3dx$

$= \lim_{b\to\infty} \left[-3\int_{-5}^{1-3b} u^{-4}\,du\right]$

$= \lim_{b\to\infty} \left[-3\left(-\frac{1}{3}u^{-3}\right)\Big|_{-5}^{1-3b}\right]$

$= \lim_{b\to\infty} \left[\frac{1}{(1-3b)^3} - \frac{1}{(-5)^3}\right]$

$= \boxed{\frac{1}{125}}$

5. $\int_1^4 \frac{1}{x^2+x-6}\,dx$

$\frac{1}{x^2+x-6} = \frac{A}{x+3} + \frac{B}{x-2}$

$1 = A(x-2) + B(x-3)$

$A = -1/5 \quad B = 1/5$

$-\frac{1}{5}\int_1^4 \frac{1}{x+3}\,dx + \frac{1}{5}\int_1^4 \frac{1}{x-2}\,dx$

↑ okay ↑ asymptote @ $x=2$

$= -\frac{1}{5}\int_1^4 \frac{1}{x+3}\,dx + \frac{1}{5}\lim_{b\to 2^-}\int_1^b \frac{1}{x-2}\,dx + \frac{1}{5}\lim_{b\to 2^+}\int_b^4 \frac{1}{x-2}\,dx$

$= -\frac{1}{5}\ln|x+3|\Big]_1^4 + \frac{1}{5}\lim_{b\to 2^-}\left[\ln|b-2| - \ln|-1|\right] + \frac{1}{5}\lim_{b\to 2^+}\left[\ln 2 - \ln|b-2|\right]$

↑ diverges $-\infty$ ↑ diverges $+\infty$

if either limit diverges, the entire integral diverges.

6. Find the average value of $f(x) = \frac{\ln(2x)}{x^2}$ on $[1, 3]$

$u = \ln 2x \quad du = \frac{1}{x}dx$

$v = \frac{-1}{x} \quad dv = x^{-2}dx$

$\int_1^3 \frac{\ln 2x}{x^2} = \frac{-\ln 2x}{x}\Big|_1^3 - \int_1^3 \frac{-1}{x^2}\,dx$

$= \frac{-\ln 2x}{x}\Big|_1^3 + \int_1^3 \frac{1}{x^2}\,dx$

$= \frac{-\ln 2x}{x}\Big|_1^3 + \frac{-1}{x}\Big|_1^3$

$= \frac{-\ln 6}{3} + \frac{\ln 2}{1} + \frac{-1}{3} + 1$

$= \ln\frac{2}{6^{1/3}} + \frac{2}{3}$

Average value

$= \frac{1}{2}\left[\ln\frac{2}{6^{1/3}} + \frac{2}{3}\right]$

$= \boxed{\ln\frac{\sqrt{2}}{\sqrt[6]{6}} + \frac{1}{3}}$