AP CALCULUS BC

15.4 - Integral Practice Day

Most common and important methods:

1. Always try to simplify an integral first and see if any "basic" antiderivatives apply. Some common techniques:
    a. Write as a polynomial and use the power rule.
    b. Separate the numerator so that you have two ratios.
    c. If the numerator's degree is greater than or equal to the denominator's → long division.
2. If that doesn't work, try u-substitution. Especially if you see a function composition.
3. Next, try integration by parts.
    a. This is most effective when *u* is a log or power function. Also effective on inverse trig functions.
4. If you see a rational function:
    a. If the denominator is factorable and the numerator has a lower degree → partial fractions
5. If you see a quadratic expression with a linear term, for which the above methods aren't working → complete the square and use u-substitution. Generally, you're doing this to get the integrand to match an inverse trig form.

"Basic Integrals" to have memorized:

$\int x^n\ dx = \frac{x^{n+1}}{n+1} + c;\ n \neq -1$	$\int \sec^2 x\ dx = \tan x + c$
$\int \frac{1}{x}\ dx = \ln x + c$	$\int \csc^2 x\ dx = -\cot x + c$
$\int e^x\ dx = e^x + c$	$\int \sec x \tan x\ dx = \sec x + c$
$\int a^x\ dx = \frac{a^x}{\ln a} + c$	$\int \csc x \cot x\ dx = -\csc x + c$
$\int \sin x\ dx = -\cos x + c$	$\int \frac{1}{\sqrt{1-x^2}}\ dx = \arcsin x + c$
$\int \cos\ dx = \sin x + c$	$\int \frac{1}{1+x^2}\ dx = \arctan x + c$

More rare methods. These are fun to know, but are much less common.

1. Sums and/or differences of squares: Use Pythagorean Identities

Expression in Integrand	Related Trig Identity	make this substitution
$1 - x^2$	$1 - \sin^2\theta = \cos^2\theta$	$x = \sin\theta$
$x^2 + 1$	$\tan^2\theta + 1 = \sec^2\theta$	$x = \tan\theta$
$x^2 - 1$	$\sec^2\theta - 1 = \tan^2\theta$	$x = \sec\theta$

2. Products of sine and cosine or of tangent and secant. Use identities to put it one of these forms:
    a. $\int(in\ terms\ of\ sinx) \cdot \cos x\, dx \leftarrow u = \sin x, du = \cos x dx$
    b. $\int(in\ terms\ of\ cosx) \cdot \sin x\, dx \leftarrow u = \cos x, -du = \sin x dx$
    c. $\int(in\ terms\ of\ tanx) \cdot \sec^2 x\, dx \leftarrow u = \tan x, du = \sec^2 x dx$
    d. $\int(in\ terms\ of\ secx) \cdot \sec x \tan x\, dx \leftarrow u = \sec x, du = \sec x \tan x\, dx$
    e. Else, it's probably necessary to use a more obscure trig identity that you don't have to memorize.

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Choose a method for each of these examples and integrate:

1. $\int \frac{x^3+5}{x^2}dx$	2. $\int \frac{x^2}{e^{2x}}dx$
3. Non-traditional Example: Use integration by parts to <u>derive</u> the formula for $\int lnx\, dx$	4. $\int_0^{\infty} e^{-x}dx$

5. $\int_0^1 x\sqrt{1+8x^2}\,dx$

6. $\int 5x\left(\sqrt{x}-x^2\right)dx$

7. $\int \frac{1}{x^2-7x+10}\,dx$

8. $\int_1^\infty \frac{x^2}{(x^3+2)^2}\,dx$

9. $\int \ln(x-3)\,dx$

10. $\int \cos 2x \cdot e^{-x}\,dx$

11. $\int_0^1 \frac{1}{\sqrt[3]{x}}\,dx$

12. $\int \frac{3x^2}{x^2+9}\,dx$