AP CALCULUS BC

Choose a method for each of these examples and integrate:

1. $\int \frac{x^3+5}{x^2}\,dx$

$= \int x\,dx + \int 5x^{-2}\,dx$

$= \frac{1}{2}x^2 + \frac{5}{-1}x^{-1} + C$

$= \frac{x^2}{2} - \frac{5}{x} + C$

2. $\int \frac{x^2}{e^{2x}}\,dx = \int x^2 e^{-2x}\,dx$

DIFF		INT
x^2	+	e^{-2x}
$2x$	−	$-\frac{1}{2}e^{-2x}$
2	+	$\frac{1}{4}e^{-2x}$
0		$-\frac{1}{8}e^{-2x}$

$= -\frac{x^2}{2}e^{-2x} - \frac{x}{2}e^{-2x} - \frac{1}{4}e^{-2x} + C$

$= -e^{-2x}\left(\frac{x^2}{2} + \frac{x}{2} + \frac{1}{4}\right) + C$

3. Non-traditional Example: Use integration by parts to <u>derive</u> the formula for $\int \ln x\,dx$

$u = \ln x \qquad du = \frac{1}{x}dx$

$v = x \qquad dv = dx$

$uv - \int v\,du$

$= x\ln x - \int x\left(\frac{1}{x}\right)dx$

$= x\ln x - x + C$

4. $\int_0^\infty e^{-x}\,dx$

$= \lim_{b\to\infty} \int_0^b e^{-x}\,dx$

$= \lim_{b\to\infty} \left(-e^{-x}\Big|_0^b\right)$

$= \lim_{b\to\infty} \left(-e^{-b} + e^0\right)$

$= \boxed{1}$

5. $\int_0^1 x\sqrt{1+8x^2}\,dx$

$u = 1+8x^2 \qquad du = 16x\,dx$

$\int_0^1 \to \int_1^9 \qquad \frac{1}{16}du = x\,dx$

$= \int_1^9 \frac{1}{16} u^{1/2}\,du$

$= \frac{1}{16}\left[\frac{2}{3}u^{3/2}\Big|_1^9\right]$

$= \frac{1}{16}\left[\frac{2}{3}(27) - \frac{2}{3}\right]$

$= \frac{1}{16}\left[18 - \frac{2}{3}\right] = \frac{9}{8} - \frac{1}{24}$

$= \cancel{\frac{1}{16}}$ $\quad = \frac{27-1}{24} = \frac{26}{24} = \boxed{\frac{13}{12}}$

6. $\int 5x(\sqrt{x} - x^2)\,dx$

$= \int 5x(x^{1/2} - x^2)\,dx$

$= \int (5x^{3/2} - 5x^3)\,dx$

$= 5(\frac{2}{5})x^{5/2} - \frac{5}{4}x^4 + C$

$= \boxed{2x^{5/2} - \frac{5}{4}x^4 + C}$

7. $\int \frac{1}{x^2-7x+10}\,dx$

$\frac{1}{x^2-7x+10} = \frac{A}{x-2} + \frac{B}{x-5}$

$1 = A(x-5) + B(x-2)$

$1/3 = B \qquad A = -1/3$

$-\frac{1}{3}\int \frac{dx}{x-2} + \frac{1}{3}\int \frac{dx}{x-5}$

$= -\frac{1}{3}\ln|x-2| + \frac{1}{3}\ln|x-5|$

8. $\int_1^\infty \frac{x^2}{(x^3+2)^2}\,dx \qquad u = x^3+2 \quad du = 3x^2dx$

$\int_1^b \to \int_3^{b^3+2}$

$\int_1^b \frac{x^2}{(x^3+2)^2}\,dx = \frac{1}{3}\int^b u^{-2}\,du$

$= -\frac{1}{3}u^{-1}\Big]_3^{b^3+2}$

$= \frac{-1}{3(b^3+2)} + \frac{1}{3\cdot 3}$

$= \boxed{\frac{1}{9}}$

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9. $\int \ln(x-3)\,dx$ PARTS!

$u = \ln(x-3)$ $du = \frac{1}{x-3}dx$

$v = x$ $dv = dx$

$= x\ln(x-3) - \int \frac{x}{x-3}\,dx$ ← I don't want to...

$u = x-3$

$\int \ln u\,du$

$= u\ln u - u + C$

$= (x-3)\ln(x-3) - (x-3) + C$

$\boxed{= (x-3)\ln(x-3) - x + C}$

10. $\int \cos 2x \cdot e^{-x}\,dx$

$u = \cos 2x$ $du = -2\sin 2x\,dx$

$v = -e^{-x}$ $dv = e^{-x}dx$

$\int \cos 2x e^{-x}dx = -\cos 2x e^{-x} - \int 2\sin 2x e^{-x}dx$

$u = \cancel{\cos 2x}\ \sin 2x$ $du = 2\cos 2x$

$v = -e^{-x}$ $dv = e^{-x}dx$

$\int \cos 2x e^{-x}dx = -\cos 2x e^{-x} - 2\left(-\sin 2x e^{-x} + \int e^{-x}2\cos 2x\,dx\right)$

$\int e^{-x}\cos 2x\,dx = -e^{-x}\cos 2x + 2e^{-x}\sin 2x - 4\int e^{-x}\cos 2x\,dx$

$\boxed{= \frac{-e^{-x}\cos 2x + 2e^{-x}\sin 2x}{5} + C}$

11. $\int_0^1 \frac{1}{\sqrt[3]{x}}\,dx$ ← asymptote!

$= \lim_{b\to 1^-} \int_0^b x^{-1/3}\,dx$

$= \lim_{b\to 1^-} \frac{3}{2}x^{\frac{2}{3}}\Big|_0^b$

$= \boxed{\frac{3}{2}}$

12. $\int \frac{3x^2}{x^2+9}\,dx$

$$\begin{array}{r} 3 \\ x^2+9\overline{)3x^2+0x+0} \\ -(3x^2+\cancel{0x}+27) \\ \hline -27 \end{array}$$

$= \int 3\,dx - 27\int \frac{1}{x^2+9}\,dx$

side note:

$\int \frac{dx}{x^2+9} = \frac{1}{9}\int \frac{dx}{(\frac{x}{3})^2+1}$

$u = \frac{x}{3}$ $3du = dx$

$= \frac{3}{9}\int \frac{du}{u^2+1}$

$= \frac{3}{9}\arctan u$

$= \frac{3}{9}\text{arc}(\frac{x}{3})$

$= 3x - 27\left(\frac{1}{3}\arctan\left(\frac{x}{3}\right)\right)$

$= 3x - 9\arctan\left(\frac{x}{3}\right) + C$