# BC Calculus

## Review #9 - Series

1996 BC2 (no calculator)

The Maclaurin series for *f(x)* is given by $1+\frac{x}{2!}+\frac{x^2}{3!}+\frac{x^4}{4!}+...+\frac{x^{2n}}{(n+1)!}+...$

a) Find $f'(0)$ and $f^{(17)}(0)$.

b) For what values of *x* does the series converge? Show your reasoning.

c) Let $g(x)=xf(x)$. Write the Maclaurin series for *g(x)*, showing the first three nonzero terms.

d) Write *g(x)* in terms of a familiar function without using series.

1997 BC2 (calculator allowed)

Let $P(x) = 7 - 3(x-4) + 5(x-4)^2 - 2(x-4)^3 + 6(x-4)^4$ be the fourth degree Taylor polynomial for the function *f* about 4. Assume that *f* has derivatives of all orders for all real numbers.

a) Find $f(4)$ and $f'''(4)$.

b) Write the second degree Taylor polynomial for $f'$ about 4 and use it to approximate $f'(4.3)$.

c) Write the fourth degree Taylor polynomial for $g(x) = \int_0^x f(t)\, dt$ about 4.

d) Can *f(3)* be determined from the information given? Justify your answer.

1999 BC4 (calculator allowed)

The function *f* has derivatives of all orders for all real numbers *x*. Assume that $f(2)=-3,\ f'(2)=5,\ f''(2)=3,$ and $f'''(2)=-8.$

a) Write the third degree Taylor polynomial for *f* about *x = 2* and use it to approximate $f(1.5)$.

b) The fourth derivative of *f* satisfies the inequality $\left|f^{(4)}(x)\right|\leq 3$ for all *x* in the closed interval *[1.5, 2]*. Use the Lagrange error bound on the approximation to *f(1.5)* to explain why $f(1.5)\neq -5$.

c) Write the fourth degree Taylor polynomial, *P(x)*, for $g(x)=f\left(x^2+2\right)$ about *x = 0*. Use *P* to explain why *g* must have a relative minimum at *x = 0*.

2003B BC6 (no calculator)

The function *f* has a Taylor series about *x* = *2* that converges to *f(x)* for all *x* in the interval of convergence. The *n*th derivative of *f* at *x* = *2* is given by $f^{(n)}(2) = \dfrac{(n+1)!}{3^n}$ for $n \geq 1,$ and $f(2) = 1.$

a) Write the first four terms and the general term of the Taylor series for *f* about *x* = *2*.

b) Find the radius of convergence for the Taylor series for *f* about *x* = *2*.

c) Let *g* be a function satisfying $g(2) = 3$ and $g'(x) = f(x)$ for all *x*. Write the first four terms and the general term of the Taylor series for *g* about *x* = *2*.

d) Does the Taylor series for *g* as defined in part c) converge at *x* = *2*? Give a reason for your answer.

2004B BC2 (no calculator)

Let *f* be a function having derivatives of all orders for all real numbers. The third degree Taylor polynomial for *f* about *x* = *2* is given by $T(x) = 7 - 9(x-2)^2 - 3(x-2)^3$.

a) Find $f(2)$ and $f''(2)$.

b) Is there enough information to determine whether *f* has a critical point at *x* = *2*?

If not, explain why not.

If so, determine whether *f(2)* is a relative maximum, a relative minimum, or neither. Justify your answer.

c) Use *T(x)* to find an approximation for *f(0)*. Is there enough information given to determine whether *f* has a critical point at *x* = *0*?

If not, explain why not.

If so, determine whether *f(0)* is a relative maximum, a relative minimum, or neither. Justify your answer.

d) The fourth derivative of *f* satisfies the inequality $\left|f^{(4)}(x)\right| \le 6$ for all *x* in the closed interval *[0, 2]*. Use the Lagrange error bound on the approximation for *f(0)* found in part c) to explain why *f(0)* is negative.

2001 BC6 (no calculator)

A function is defined by $f(x)=\frac{1}{3}+\frac{2}{3^2}x+\frac{3}{3^3}x^2+\ldots+\frac{n+1}{3^{n+1}}x^n+\ldots$ for all $x$ in the interval of convergence of the given power series.

a) Find the interval of convergence for this power series. Show the work that leads to your answer.

b) Find $\lim_{x\to 0}\frac{f(x)-\frac{1}{3}}{x}$.

c) Write the first three nonzero terms and the general term for an infinite series that represents $\int_0^1 f(x)\,dx$.

d) Find the sum of the series determined in part c).