BC Calculus

Review #4 – Differential Equations

1988 BC6 (no calculator)

Let *f* be a differentiable function for all $x \geq 0$ such that $f(0) = 5$ and $f(3) = -1$.

Suppose that for any number $b > 0$ the average value of *f(x)* on the interval $0 \leq x \leq b$ is $\frac{f(0) + f(b)}{2}$.

a) Find $\int_0^3 f(x)\,dx$.

b) Prove that $f'(x) = \frac{f(x) - 5}{x}$ for all $x > 0$.

c) Find $f(x)$.

1990 BC6 (no calculator)

Let *f* and *g* be continuous functions with the following properties.

i) $g(x) = A - f(x)$ where *A* is a constant.

ii) $\int_1^2 f(x)\,dx = \int_2^3 g(x)$.

iii) $\int_2^3 f(x)\,dx = -3A$.

a) Find $\int_1^3 f(x)\,dx$ in terms of *A*.

b) Find the average value of *g(x)* in terms of *A,* over the interval [1, 3].

c) Find the value of *k* if $\int_0^1 f(x+1)\,dx = kA$.

1992 AB6 (no calculator)

At time *t*, $t \geq 0,$ the volume of a sphere is increasing at a rate proportional to the reciprocal of its radius. At $t = 0,$ the radius of the sphere is 1 and at $t = 15,$ the radius of the sphere is 2. (the volume of a sphere is given by $V = \frac{4}{3}\pi r^3$).

a) Find the radius as a function of *t*.

b) At what time *t* will the volume of the sphere be 27 times its volume at $t = 0$?

1998 AB4 (calculator)

Let *f* be a function with $f(1) = 4$ such that for all points *(x, y)* on the graph of *f*, the slope is given by $\frac{3x^2 + 1}{2y}$.

a) Find the slope of the graph of *f* at the point where $x = 1$.

b) Write an equation for the line tangent to the graph of *f* at $x = 1$ and use it to approximate *f(1.2)*.

c) Find *f(x)* by solving the separable differential equation $\frac{3x^2 + 1}{2y}$ with the initial condition that $f(1) = 4$.

d) Use your solution from part c) to find *f(1.2)*.