BC Calculus

Review #2 - Derivatives

Differentiate the following

1) $f(x) = x^{14} + 10x^2 + 7x - 4$

2) $y = x^{-2} - 3x^{-5} + 3x^{-8}$

3) $g(x) = \dfrac{x^3 - 3x + 5}{x^2}$

4) $f(x) = \cos\left(\dfrac{x-1}{x+1}\right)$

5) $k(x) = \dfrac{(3x-2)^6}{(2x+1)^7}$

6) $f(x) = \ln\left(x^{\frac{3}{2}} + 2x^{\frac{3}{5}} - 4x^{\frac{4}{7}}\right)$

7) $F(x) = \arctan(4x+3)$

8) $f(x) = \sin^3\left(3x^2\right)e^{-4x}$

9) $f(x) = \cot(3x)\sec^4(2x)$

10) $y = (\sin x)^{\tan x}$

For #15-16, Find dy/dx

11) $x^2 - \ln(2xy) + 3y^2 = 2$

12) $y^2 + (3\cos x)\, y^2 + 3x^3 - 5 = 0$

13) Write an equation for the lines tangent to and normal to the graph of $4x^2 + 9y^2 = 36$ when $x = 6$.

14) Given the parametric function $x = 3t - 5$ and $y = t^2 + 2t - 4$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

1992 BC3 (calculator active)

At time t, $0 \le t \le 2\pi$, the position of a particle moving along a path in the xy-plane is given by the parametric equations $x = e^t \sin t$ and $y = e^t \cos t$.

a) Find the slope of the path of the particle when $t = \frac{\pi}{2}$.

b) Find the speed of the particle when $t = 1$.

c) Find the distance traveled by the particle along the path from $t = 0$ to $t = 1$.

1996 AB4 BC4 (calculator allowed)

This problem deals with functions by $f(x) = x + b\sin x,$ where *b* is a positive constant and $-2\pi \le x \le 2\pi.$

a) Sketch the graph of the two functions, $y = x + \sin x$ and $y = x + 3\sin x.$

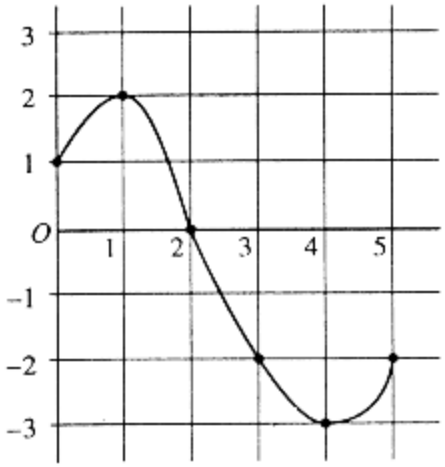
b) Find the *x*-coordinates of all points, $-2\pi \le x \le 2\pi,$ where the line $y = x + b$ is tangent to the graph of $f(x) = x + b\sin x.$

c) Are the points of tangency described in part b) relative maximum points of *f*? Why?

d) For all values of $b > 0,$ show that all inflection points of the graph of *f* lie on the line $y = x.$

1995 BC6

Let *f* be a function whose domain is the closed interval [0, 5]. The graph of *f* is shown below.



Graph of *f*

Let $h(x) = \int_0^{\frac{x}{2}+3} f(t)\,dt$.

a) Find the domain of *h*.

b) Find $h'(2)$.

c) At what *x* is $h(x)$ a minimum? Show the analysis that leads to your conclusion.

1991 BC4 (no calculator)

A particle moves along the *x*-axis so that at time *t* its position is given by $x(t) = \sin\left(\pi t^2\right)$ for $-1 \le t \le 1$.

a) Find the velocity at time *t*.

b) Find the acceleration at time *t*.

c) For what values of *t* does the particle change direction?

d) Find all values of *t* for which the particle is moving left.

1994 BC4 (no calculator)

A particle moves along the *x*-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = t \ln t - t$. At time $t = 1$, the position of the particle is $x(1) = 6$.

a) Write an expression for the acceleration of the particle.

b) For what values of *t* is the particle moving to the right?

c) What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.

d) Write an expression for the position, *x(t)*, of the particle.