BC Calculus

Review #1: Limits and Continuity

Determine the limit of each of the following

1. $\lim_{x\to 2}\dfrac{x-2}{x-2}$

2. $\lim_{x\to 5}\ 2x^2-4x+7$

3. $\lim_{x\to 2}\dfrac{x^2+x-6}{x-2}$

4. $\lim_{x\to -\infty}\dfrac{2x+3}{1-x^2}$

5. $\lim_{x\to 9}\dfrac{x-0}{\sqrt{x}-3}$

6. $\lim_{x\to 0}\dfrac{\sin x}{x}$

7. $\lim_{x\to 2}\dfrac{x}{4-x^2}$

8. $\lim_{x\to \infty}\dfrac{x^2+4}{x-x^2}$

9. Which of the following is a horizontal asymptote for $f(x)=\dfrac{6x^2+2x-4}{2x^2+3x+2}$?

   a) $y=-3$ b) $y=-2$ c) $y=2$ d) $y=3$ e) $y=4$

10. What is the $\lim_{x\to 3^+}\dfrac{x+3}{x-3}$?

11. Find $\lim_{x\to -\infty}\dfrac{|8x+6|}{4x-2}$

12. Let $f(x)=\begin{cases} x^2-2, & x<1 \\ -\dfrac{1}{2}x+1, & x\geq 1 \end{cases}$. Find

    a) $\lim_{x\to 1^+} f(x)$ b) $\lim_{x\to 1^-} f(x)$ c) $\lim_{x\to 1} f(x)$

13. Rank each kind of function in increasing order according to its order of magnitude.

    a) power function: $f(x)=x^n$

    b) logarithmic function: $g(x)=\ln x$

    c) exponential function: $h(x)=7^x$

    d) Factorial function: $k(x)=x!$

14. The graph of *f* is shown below. At which of the following points is *f* continuous?

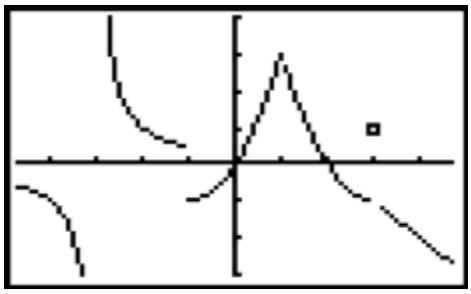


Figure 17

a) $x=-3$

b) $x=-1$

c) $x=1$

d) $x=3$

e) all of the above

15. The figure shown in the problem above has a removable discontinuity at which of the following points?

a) $x=-3$ b) $x=-1$ c) $x=0$ d) $x=1$ e) $x=3$

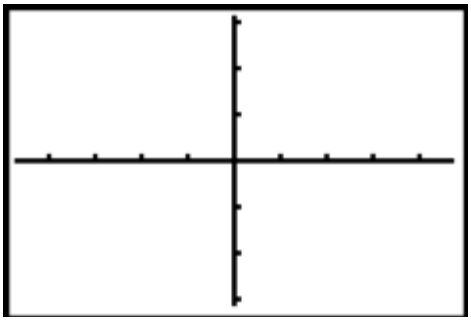
16. Sketch the graph of a function with the following properties.

i) $f(-2)$ exists

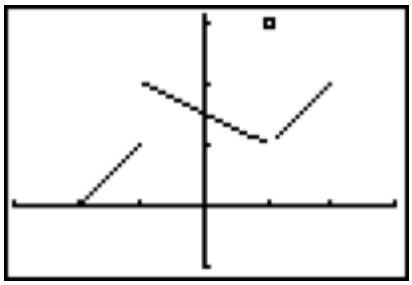
ii) $\lim_{x\to -2}$ exists

iii) *f* is not continuous at $x=2$

iv) $\lim_{x\to 1} f(x)$ does not exist



17. Let $y=f(x)$ be the function shown below. Which of the following statements is false?



a) $\lim_{x\to 1} f(x)=1$

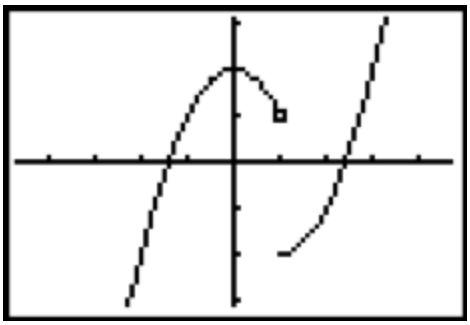
b) $\lim_{x\to 2^-} f(x)=2$

c) $\lim_{x\to 0^+} f(x)=\lim_{x\to 0^-} f(x)$

d) $\lim_{x\to -1} f(x)=2$

18. How many times do the graphs of $y=2^x$ and $y=x^{100}$ intersect?

19. Use the graph below to answer the following questions.



Graph of g(x)

a) $\lim_{x\to 1^-} g(x)$ =

b) $\lim_{x\to 1^+} g(x)$ =

c) $\lim_{x\to -1} g(x)$ =

d) $g(1) =$

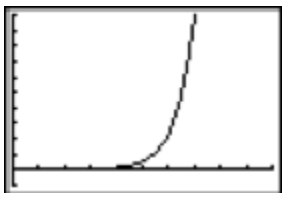
20. Without using L'Hopital's Rule, evaluate each of the following. Show your work.

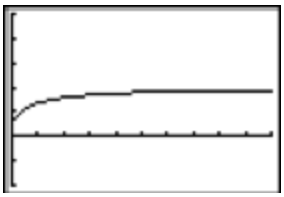
a) $\lim_{x\to\infty} \frac{\ln 3x}{x^3}$

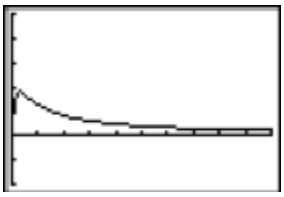
b) $\lim_{x\to\infty} \frac{x^{100}}{e^{0.01x}}$

c) $\lim_{x\to\infty} \frac{\sqrt{x}}{x}$

21. The graph of $\frac{f(x)}{g(x)}$ is shown for each of the following. State whether $f(x)$ or $g(x)$ has the higher order of magnitude.

a) 

b) 

c) 

22. CALCULATOR ALLOWED: The graph of $y = 2^x$ and $y = x^2$ are shown below.

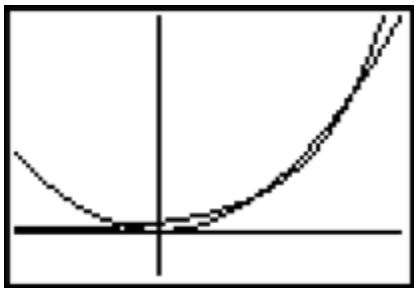


Figure 25

a) Find the coordinates of all three points of intersection.

b) Find the area of the regions bounded by the two graphs.

23. Let *f* be the function defined by $f(x) = 2xe^{2x}$

a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$

c) Find the absolute minimum value of *f*. Justify your answer.

c) What is the range of *f*?