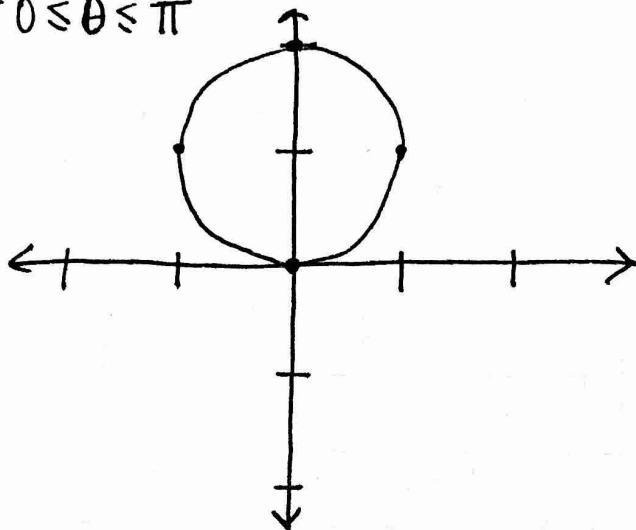


11-76

$$r = 2 \sin \theta \text{ for } 0 \leq \theta \leq \pi$$

$\theta$	$r$
0	0
$\pi/4$	$\sqrt{2}$
$\pi/2$	2
$3\pi/4$	$\sqrt{2}$
$\pi$	0



a)

$\theta$	Slope estimate
0	0 - horizontal
$\pi/4$	undefined - vertical
$\pi/2$	0 - horizontal
$3\pi/4$	undefined - vertical

b)  $r = 2 \sin \theta$

$$r^2 = 2r \sin \theta$$

$$x^2 + y^2 = 2y$$

$$x^2 + y^2 - 2y + 1 = 0 + 1$$

$$\boxed{x^2 + (y-1)^2 = 1}$$

Derivative:

$$\frac{d}{dx}(x^2 + (y-1)^2) = \frac{d}{dx}(1)$$

$$2x + 2(y-1) \frac{dy}{dx} = 0$$

$$2(y-1) \frac{dy}{dx} = -2x$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y-1}}$$

c)  $\theta = 0 \rightarrow \frac{dy}{dx} = \frac{-0}{0-1} = 0$   
 (0,0)

$\theta = \pi/4 \rightarrow \frac{dy}{dx} = \frac{-1}{1-1} = \frac{-1}{0}$  undefined  
 (1,1)

$\theta = \pi/2 \rightarrow \frac{dy}{dx} = \frac{-0}{2-1} = 0$   
 (0,2)

$\theta = 3\pi/4 \rightarrow \frac{dy}{dx} = \frac{1}{1-1} = \frac{1}{0}$  undefined.  
 (-1,1)

11-76d)

step 1: convert the equation to rectangular form

step 2: use implicit differentiation to find a formula for  $\frac{dy}{dx}$ 

11-77a)

$r = \theta \dots$

$r^2 = \theta^2 ?$   
no

$r^2 = \theta r ?$   
no

IMPOSSIBLE TO WRITE IN RECTANGULAR FORM

b)

 $\frac{dr}{d\theta}$  gives the change in  $r$  (distance from pole) as  $\theta$  changes (moving counterclockwise).This is different from  $\frac{dy}{dx}$ 

11-78

a)

$x = r \cos \theta \quad y = r \sin \theta$

$r = \theta$

$x = \theta \cos \theta \quad y = \theta \sin \theta \leftarrow \text{parametric form of the polar equation}$

b)

$\frac{dx}{d\theta} = \cos \theta + \theta(-\sin \theta)$

$\frac{dy}{d\theta} = \sin \theta + \theta \cos \theta$

$\frac{dy}{dx} = \frac{\sin \theta + \theta \cos \theta}{\cos \theta - \theta \sin \theta}$

c)

$\theta = 0 \rightarrow \frac{dy}{dx} = \frac{0+0}{1-0} = 0$

$\theta = \pi/2 \rightarrow \frac{dy}{dx} = \frac{1 + \pi/2(0)}{0 - \pi/2(1)} = \frac{1}{-\pi/2} = -\frac{2}{\pi}$

$\theta = \pi \rightarrow \frac{dy}{dx} = \frac{0 + \pi(-1)}{-1 - \pi(0)} = \frac{-\pi}{-1} = \pi$

d)

convert to parametric form, then compute  $\frac{dy}{dx}$  using  $\frac{dy/d\theta}{dx/d\theta}$ 

→

11-79 a)  $r=6$

$$r^2 = 36$$

$$x^2 + y^2 = 36$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

b)  $r = 3 \sin \theta$

$$r^2 = 3r \sin \theta$$

$$x^2 + y^2 - 3y = 0$$

$$2x + 2y \frac{dy}{dx} - 3 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{2y-3}$$

parametric version:

$$x = r \cos \theta \quad y = r \sin \theta$$

c) ~~parametric~~

$$2r \sin \theta + 5r \cos \theta = 10$$

$$2y + 5x = 10$$

$$y = \frac{-5}{2}x + 5$$

$$\boxed{\frac{dy}{dx} = \frac{-5}{2}}$$

~~or~~  $r = 3 \sin \theta$

$$x = 3 \sin \theta \cos \theta \quad y = 3 \sin^2 \theta$$

$$\frac{dx}{d\theta} = 3(\cos^2 \theta - \sin^2 \theta) \quad \frac{dy}{d\theta} = 6 \sin \theta \cos \theta$$

$$\frac{dy}{dx} = \frac{6 \sin \theta \cos \theta}{3(\cos^2 \theta - \sin^2 \theta)}$$

$$= \frac{3 \cdot \sin 2\theta}{3 \cos 2\theta}$$

$$= \boxed{\tan 2\theta}$$

d)  $r = 2\theta$

$$x = r \cos \theta \quad y = r \sin \theta$$

$$x = 2\theta \cos \theta \quad y = 2\theta \sin \theta$$

$$\frac{dx}{d\theta} = 2[\cos \theta - \theta \sin \theta] \quad \frac{dy}{d\theta} = 2[\sin \theta + \theta \cos \theta]$$

$$\boxed{\frac{dy}{dx} = \frac{\sin \theta + \theta \cos \theta}{\cos \theta - \theta \sin \theta}}$$

== REVIEW/PREVIEW ==

11-80  $r = -1 + \cos \theta$ ,  $\theta = \pi/2$  tangent line slope  $\frac{dy}{dx}$  point  $(x, y)$



11-80

$$r = -1 + \cos\theta$$

$$x = r\cos\theta \quad y = r\sin\theta$$

$$x = (-1 + \cos\theta)\cos\theta \quad y = (-1 + \cos\theta)\sin\theta$$

$$x = -\cos\theta + \cos^2\theta \quad y = -\sin\theta + \sin\theta\cos\theta$$

$$\frac{dx}{d\theta} = \sin\theta + 2\cos\theta(-\sin\theta) \quad \frac{dy}{d\theta} = -\cos\theta + \cos^2\theta - \sin^2\theta$$

$$\frac{dy}{dx} = \frac{-\cos\theta + \cos^2\theta - \sin^2\theta}{\sin\theta - 2\cos\theta\sin\theta}$$

$$\theta = \pi/2 \rightarrow \frac{dy}{dx} = \frac{-0 + 0 - 1}{1 - 2(0)(1)} = -1 \leftarrow \text{slope}$$

$$R = -1 + \cos\left(\frac{\pi}{2}\right) = -1$$

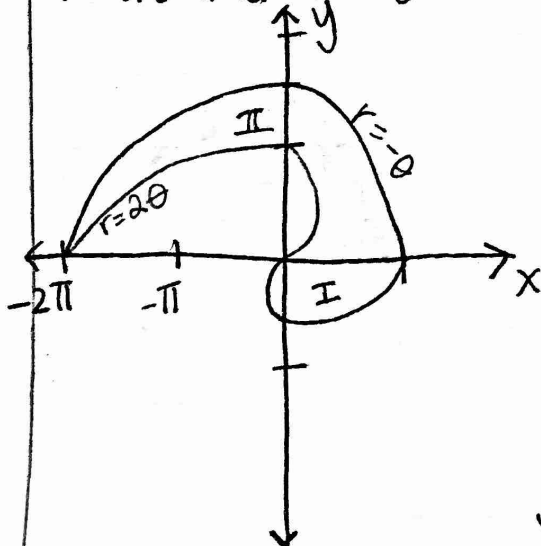
$$x = -1 \cos\left(\frac{\pi}{2}\right) = 0 \leftarrow \text{point } (0, -1)$$

$$y = -1 \sin\left(\frac{\pi}{2}\right) = -1$$

$$y + 1 = -1(x - 0)$$

$$y + 1 = -x$$

$$\boxed{y = -x - 1}$$

11-81  $r = 2\theta$  and  $r = -\theta$ region I:  $r = -\theta$  from pole to axis

$$\theta = 0 \text{ to } \theta = \pi$$

$$\int_0^{\pi} \frac{1}{2}(-\theta)^2 d\theta = \frac{\theta^3}{2 \cdot 3} \Big|_0^{\pi} = \frac{\pi^3}{6}$$

region II: outer:  $r = -\theta$  from  $\theta = \pi$  to  $\theta = 2\pi$ inner:  $r = 2\theta$  from  $\theta = 0$  to  $\theta = \pi$ 

$$\int_{\pi}^{2\pi} \frac{1}{2}\theta^2 d\theta - \int_0^{\pi} \frac{1}{2}(2\theta)^2 d\theta = \left(\frac{(2\pi)^3}{6} - \frac{\pi^3}{6}\right) - \frac{4\pi^3}{6}$$

$$\text{Total: } \frac{4\pi^3}{6} \approx 20.671$$

11-82 a)  $\int_0^2 \frac{dx}{(x-2)^2} \leftarrow \text{vertical asymptote @ } x=2$

$$= \lim_{b \rightarrow 2^-} \left( \int_0^b \frac{dx}{(x-2)^2} \right)$$

$$= \lim_{b \rightarrow 2^-} \left( \frac{-1}{x-2} \Big|_0^b \right)$$

$$= \lim_{b \rightarrow 2^-} \left( \frac{-1}{b-2} - \frac{-1}{-2} \right)$$

$$= \boxed{\infty \leftarrow \text{diverges}}$$

b)  $\int \sec^2 x \ln(\tan x) dx$   $u = \tan x$   $du = \sec^2 x dx$

$$= \int \ln w dw \text{ parts! } u = \ln w \quad dv = dw$$

$$du = \frac{dw}{w} \quad v = w$$

$$= w \ln w - \int dw$$

$$= w \ln w - w$$

$$= \boxed{\tan x \ln(\tan x) - \tan x + C}$$

c)  $\int \frac{3}{(x+1)(x+2)} dx \rightarrow \frac{3}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$$3 = A(x+2) + B(x+1)$$

$$x = -2 \rightarrow 3 = -3B$$

$$-1 = B$$

$$x = 1 \rightarrow 3 = A(3)$$

$$1 = A$$

$$= \int \frac{dx}{x+1} - \int \frac{dx}{x+2}$$

$$= \boxed{\ln \left| \frac{x-1}{x+2} \right| + C}$$

→

11-82d)  $\int 6x \tan(x^2) dx$        $u = x^2$      $du = 2x dx$   
 $3du = 6x dx$

$= 3 \int \tan u du$

$= 3 \int \frac{\sin u}{\cos u} du$      $w = \cos u$      $dw = -\sin u du$

$= -3 \int \frac{dw}{w}$

$= -3 \ln|w| + C$

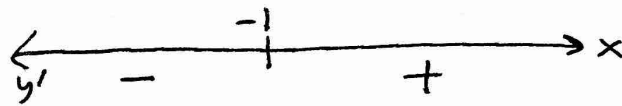
$= -3 \ln|\cos u| + C$

$= \boxed{-3 \ln|\cos(x^2)| + C}$

11-83  $y = xe^x$        $x\text{-int: } 0 = xe^x$        $y\text{-int: } y = 0 \cdot e^0$   
 $\boxed{x=0}$        $\boxed{y=0}$

$y' = e^x + xe^x$

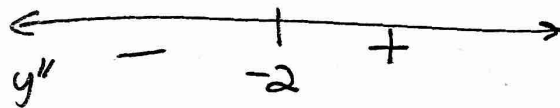
$y' = 0 \rightarrow e^x(1+x) = 0$   
 $x = -1$



decreasing on  $(-\infty, -1)$   
 increasing on  $(-1, \infty)$   
 relative ~~max~~ min @  $x = -1$

$y'' = e^x + e^x + xe^x$

$y'' = 0 \rightarrow e^x(2+x) = 0$   
 $x = -2$



Concave down on  $(-\infty, -2)$

Concave up on  $(-2, \infty)$

Point of inflection at  $x = -2$

11-84  $s = -\ln\left(\frac{1}{t+1}\right)$  meters is the distance traveled after  $t$  mins.

a) avg velocity on  $0 \leq t \leq 3$

$$v = \frac{ds}{dt} \quad \text{avg velocity} = \frac{\Delta s}{\Delta t} = \frac{s(3) - s(0)}{3 - 0} \approx 0.462 \text{ m/min}$$

b) acceleration @  $t=2$

$$s = -\ln(t+1)^{-1} = \ln(t+1)$$

$$\frac{ds}{dt} = v = \frac{1}{t+1}$$

$$\frac{dv}{dt} = a = \frac{-1}{(t+1)^2}$$

$$a(2) = \frac{-1}{3^2} = \boxed{-\frac{1}{9} \text{ m/min}^2}$$

11-85 @  $x=2$ ,  $f(2) > 0$ ,  $f'(2) = 0$ ,  $f''(2) < 0$

$$\text{so } f''(2) < f'(2) < f(2) \quad \boxed{E}$$

11-86 for  $t > 0$ ,  $x = (2t-1)^4$ ,  $y = t^2 + 1$

$$\vec{v} = \langle 4 \cdot 2 \cdot (2t-1)^3, 2t \rangle$$

$$\vec{a} = \langle 4 \cdot 3 \cdot 2^2 (2t-1)^2, 2 \rangle$$

$$\vec{a}|_{t=1} = \langle 48, 2 \rangle \quad \boxed{E}$$

11-87  $\frac{x}{x-1} + xy = y^2 - 3x$  slope @  $(2, 4)$

$$(x-1)^{-1} + x(-1)(x-1)^{-2} + y + x \frac{dy}{dx} = 2y \frac{dy}{dx} - 3$$

$$(2-1)^{-1} + 2(-1)(2-1)^{-2} + 4 + 2y' = 2(4)y' - 3$$
$$1 - 2 + 4 + 2y' = 8y' - 3$$

$$6 = 6y' \quad \boxed{C}$$
$$y' = 1$$

11-88

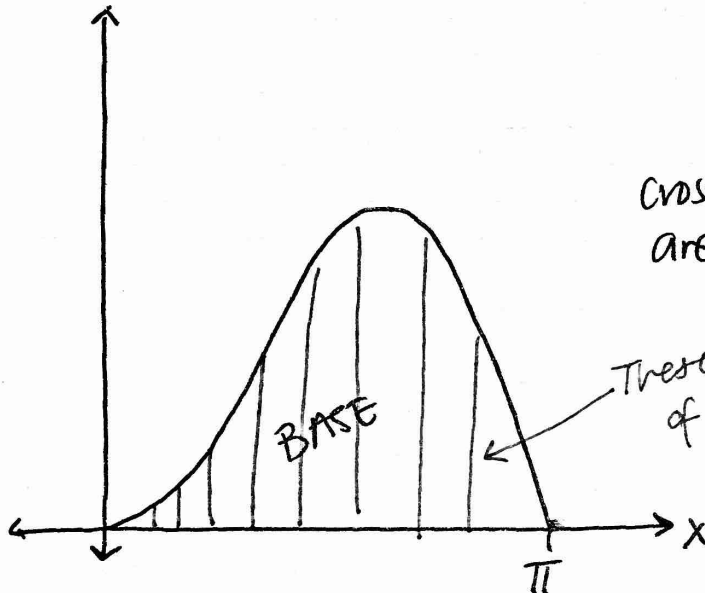
radius of convergence for  $\sum_{n=1}^{\infty} n! x^n$ 

$$\lim_{n \rightarrow \infty} \left| \frac{(n+1)! x^{n+1}}{n! x^n} \right| = \lim_{n \rightarrow \infty} |x(n+1)| = \infty \leftarrow \text{never finite not } < 1$$

Converges never  
radius = 0**B**

11-89

$y = x \sin x$

x-axis on  $0 \leq x \leq \pi$ cross sections  $\perp$  to x-axis  
are semicirclesThese are the diameters  
of the semicircles

$2r = x \sin x$

$r = \frac{x \sin x}{2}$

semicircle =  $\frac{\pi r^2}{2}$

$= \frac{\pi}{2} \left( \frac{x \sin x}{2} \right)^2$

Add up semicircles on  $0 \leq x \leq \pi$ :

$$\frac{\pi}{8} \int_0^{\pi} (x \sin x)^2 dx \approx \boxed{1.721} \quad \boxed{C}$$



11-90

a)  $r = 5 \cos \theta$

$r^2 = 5r \cos \theta$

$x^2 + y^2 = 5x$

$x^2 - 5x + y^2 = 0$

$2x - 5 + 2yy' = 0$

$2yy' = -2x + 5$

$$y' = \frac{-2x + 5}{2y}$$

$x = (5 \cos \theta) \cos \theta$

$y = (5 \cos \theta) \sin \theta$

$\frac{dy}{d\theta} = 5(-\sin^2 \theta + \cos^2 \theta)$

$$\frac{dx}{d\theta} = 5 \cdot 2 \cos \theta (-\sin \theta)$$

$$\frac{dy}{dx} = \frac{\cos(2\theta)}{-\sin(2\theta)}$$

$$\frac{dy}{dx} = -\cot(2\theta)$$

b)  $r = \frac{2}{\sqrt{\cos \theta \sin \theta}}$

$r^2 = \frac{4}{\cos \theta \sin \theta}$

$(r \cos \theta)(r \sin \theta) = 4$

$xy = 4$

$y + xy' = 0$

$y' = -\frac{y}{x}$

$xy = 4$

$y = 4/x$

$y' = -\frac{y}{x}$

$y' = -\frac{4/x}{x}$

$$y' = -\frac{4}{x^2}$$

c)  $r = 2 + 3 \cos \theta$

$x = (2 + 3 \cos \theta) \cos \theta$   ~~$f(2 \cos \theta)$~~   $y = (2 + 3 \cos \theta) \sin \theta$

$\frac{dx}{d\theta} = (-3 \sin \theta) \cos \theta + (2 + 3 \cos \theta)(-\sin \theta) = -2 \sin \theta - 6 \sin \theta \cos \theta$

$\frac{dy}{d\theta} = (-3 \sin \theta) \sin \theta + (2 + 3 \cos \theta) \cos \theta = 2 \cos \theta + 3(\cos^2 \theta - \sin^2 \theta)$

$$\frac{dy}{dx} = \frac{2 \cos \theta + 3 \cos(2\theta)}{-2 \sin \theta - 3 \sin(2\theta)}$$

11-90 d)  $r^2 = 4\sin(2\theta)$

$$r^2 \cdot r = 4r\sin(2\theta)$$

$$r^2 \cdot r = 8r\sin\theta\cos\theta$$

$$r^2 \cdot r^2 = 8r\sin\theta r\cos\theta$$

$$(x^2 + y^2)^2 = 8xy$$

$(2)(x^2 + y^2)(2x + 2yy')$  ← I don't like this

$$x^4 + 2x^2y^2 + y^4 = 8xy$$

$$4x^3 + 2(2xy^2 + x^2 2yy') + 4y^3y' = 8(y + xy')$$

$$4x^3 + 4xy^2 + 4x^2yy' + 4y^3y' = 8y + 8xy'$$

$$y'(4x^2y + 4y^3 - 8x) = 8y - 4x^3 - 4xy^2$$

$$y' = \frac{-4xy^2 - 4x^3 + 8y}{4x^2y + 4y^3 - 8x}$$

11-91  $r_1 = 1 + \cos\theta$   $r_2 = 5(1 + \cos\theta)$  ← cardioids of different sizes

a) Since the radius is just bigger by a factor of 5,  $r_2$  is just a dilation ... 5 times bigger.

So, for any angle  $\theta$ , the slope  $\frac{dy}{dx}$  should be the same for both graphs.

b)  $r = a(1 + \cos\theta)$  find  $\frac{dy}{dx}$

same as  $\frac{dy}{dx}$  for  $r = 1 + \cos\theta$

$$x = (1 + \cos\theta)\cos\theta$$

$$y = (1 + \cos\theta)\sin\theta$$

$$\frac{dy}{d\theta} = -\sin^2\theta + \cos^2\theta + \cos\theta$$

$$\frac{dx}{d\theta} = -2\sin\theta\cos\theta - \sin\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos\theta + \cos(2\theta)}{-\sin\theta - \sin(2\theta)}$$

11-91 c) For any value of  $\theta$ , the tangent slopes are the same for  
 $r = 1 + \cos \theta \quad \& \quad r = a(1 + \cos \theta)$

11-92  $r = 1 + \cos \theta \quad r = 5(1 + \cos \theta)$

Smaller area:  $\int_0^{2\pi} \frac{1}{2} (1 + \cos \theta)^2 d\theta$  When does  $1 + \cos \theta = 0 \leftarrow$  cross pole  
 $\cos \theta = -1$

$$\int_0^{2\pi} \frac{1}{2} (1 + \cos \theta)^2 d\theta$$

$$\theta = \pi + 2\pi k$$

$$\theta = \{ \pi, 3\pi, 5\pi, \dots \}$$

~~$\int_0^{2\pi}$~~  calculator  $\approx \underline{4.712} \leftarrow$  smaller

Larger area:

$$\int_0^{2\pi} \frac{1}{2} (5(1 + \cos \theta))^2 d\theta \approx \underline{117.810} \leftarrow \text{larger.}$$

The ratio is 25:1

in general, ~~the~~

$r = a(1 + \cos \theta)$  will have  $a^2$  times as much area  
as  $r = 1 + \cos \theta$

(This makes sense b/c the radii are @ an  $a:1$   
ratio.)

11-93  $r = e^{\theta/10}$

a)  $\theta = \pi/3 \leftarrow$  crosses this ray at  $r = e^{\pi/3/10} = e^{\pi/30}$

and  $r = e^{7\pi/3/10} = e^{7\pi/30}$

and  $r = e^{13\pi/3/10} = e^{13\pi/30}$

Continue d...  $\longrightarrow$

11-93

a)  $r = e^{\theta/10}$

$x = e^{\theta/10} \cos \theta$

$y = e^{\theta/10} \sin \theta$

$$\frac{dx}{d\theta} = \frac{1}{10} e^{\theta/10} \cos \theta - e^{\theta/10} \sin \theta \quad \frac{dy}{d\theta} = \frac{1}{10} e^{\theta/10} \sin \theta + e^{\theta/10} \cos \theta$$

$$\frac{dy}{dx} = \frac{e^{\theta/10} (\frac{1}{10} \sin \theta + \cos \theta)}{e^{\theta/10} (\frac{1}{10} \cos \theta - \sin \theta)}$$

$$\frac{dy}{dx} = \frac{\sin \theta + 10 \cos \theta}{\cos \theta - 10 \sin \theta}$$

When  $\theta = \pi/3$  or any coterminal angle ...

$$\frac{dy}{dx} = \frac{\frac{\sqrt{3}}{2} + 5}{\frac{1}{2} - 5\sqrt{3}} = \frac{\sqrt{3} + 10}{1 - 10\sqrt{3}} \approx -0.719$$

The slopes are the same for any point crossing  $\theta = \pi/3$   
(the ray)

b) THE SLOPES ARE THE SAME.

The derivative formula gives the same value for all coterminal sets of angles!

$$\sin(\pi/3) = \sin(7\pi/3) \text{ etc...}$$

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REVIEW/PREVIEW

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11-94 For  $0 \leq t \leq 1$ , position is  $x(t) = \frac{1}{4} e^{8t} - 2t$   $y(t) = e^{4t}$ When  $t = 1/4$ , find velocity & speed:

velocity:  $x'(t) = 2e^{8t} - 2$   $y'(t) = 4e^{4t}$

$x'(1/4) = 2e^2 - 2$   $y'(1/4) = 4e$

$$\boxed{\text{velocity: } \langle 2e^2 - 2, 4e \rangle}$$

Speed: hypotenuse!

$$= \sqrt{(2e^2 - 2)^2 + (4e)^2}$$

$$= \sqrt{4e^4 - 8e^2 + 4 + 16e^2}$$

$$= \sqrt{(2e^2 + 2)^2}$$

$$= \boxed{2e^2 + 2}$$

→

11-95  $x(t) = t^2 - t$   $y(t) = \frac{1}{3}t^3 - t$  on  $0 \leq t \leq 1$ .

Find arclength.

$$\text{Arclength} = \int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2} dt$$

$$= \int_0^1 \sqrt{(2t-1)^2 + (t^2-1)^2} dt$$

unnecessary  $\rightarrow$   $= \int_0^1 \sqrt{4t^2 - 4t + 1 + t^4 - 2t^2 + 1} dt$

calculator

$$\approx 0.903 \text{ units}$$

11-96 I)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+2}$  conv. by A.S.T.

II)  $\sum_{n=1}^{\infty} n! e^{-n} = \sum_{n=1}^{\infty} \frac{n!}{e^n}$  div. by  $n^{\text{th}}$  term test A

III)  $\sum_{n=1}^{\infty} \frac{3n}{(-1)^n}$  div. by  $n^{\text{th}}$  term test

11-97 a)  $\int_{-1}^1 x^{-1/3} dx$

$$= \frac{3}{2} x^{2/3} \Big|_{-1}^1$$

$$= \frac{3}{2} - \frac{3}{2}$$

$$= \boxed{0}$$

b)  $\int \cos x e^{\sin x} dx$

$$u = \sin x \quad du = \cos x dx$$

$$= \int e^u du$$

$$= e^u + C$$

$$= \boxed{e^{\sin x} + C}$$

c)  $\int_{-6}^6 \sin(x^3) dx$   $\leftarrow$  This is gross, but at least I know that  $\sin(x^3)$  is odd, which means that

$$\int_{-a}^a \sin(x^3) dx = \underline{\underline{0}}$$

11-97 d)  $\int \frac{\cos x}{\sin x} dx$   $u = \sin x$   $du = \cos x dx$

$$= \int \frac{du}{u}$$

$$= \ln|u| + C$$

$$= \boxed{\ln|\sin x| + C}$$

11-98

Time	9a	11a	12p	1p	2p	4p	5p	7p
Hburgs	3	39	81	251	341	498	611	894

a) @ 10 am,  $\frac{dH}{dt} \approx \frac{39-3}{11-9} = \frac{36}{2} = \underline{18 \text{ hamburgers/hr}}$

~~@ 4 pm,  $\frac{dH}{dt} \approx \frac{341-39}{2-11} = \frac{302}{-9} \approx -33.56$~~

b) from 12pm to 1pm, the rate was biggest.

11-99  $\frac{dP}{dt} = \frac{1}{160} P(40-P)$  ← logistic! lions! per year!

$t=0 \dots P(0)=12$ . Find  $P(3)$ .

$$\int \frac{dP}{P(40-P)} = \int \frac{dt}{160}$$

$$\frac{1}{40} \int \frac{dP}{P} - \frac{1}{40} \int \frac{dP}{40-P} = \int \frac{dt}{160}$$

$$4 \ln|P| - 4 \ln|40-P| = t + C, \quad P(0)=12$$

$$4 \ln \left| \frac{12}{40-12} \right| = 0 + C$$

$$C = 4 \ln \left| \frac{12}{28} \right|$$

when  $t=3 \dots 4 \ln \left| \frac{P}{40-P} \right| = 3 + 4 \ln \left| \frac{12}{28} \right|$

$$\ln \left| \frac{P}{40-P} \right| = \frac{3}{4} + \ln \left| \frac{12}{28} \right|$$

$$\left| \frac{P}{40-P} \right| = e^{3/4} \cdot \left( \frac{12}{28} \right)$$

$$P = 40 \left( \frac{12}{28} \right) e^{3/4} - P \left( \frac{12}{28} \right) e^{3/4}$$

$$P = \frac{40 \left( \frac{12}{28} \right) e^{3/4}}{1 + \left( \frac{12}{28} \right) e^{3/4}}$$

$$P \approx 19.028$$

$$\boxed{19 \text{ lions!}}$$

11-100

$$f(x) = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362880} - \dots \text{ near } x=0$$

$$= \frac{x^1}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots$$

$$= \sinh x$$

C

11-101

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n!}$$

$$p = \left| \frac{(x-1)^{n+1}}{(n+1)!} \frac{n!}{(x-1)^n} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{x-1}{n+1} \right|$$

$$= 0 < 1 \text{ always}$$

$(-\infty, \infty)$

11-102

M = avg value of  $f(x) = 3 \cdot 2^{x-1}$  on  $0 \leq x \leq 5$ .

When (for what value  $x=c$ ) does  $f(c) = M$ ?

M.V.T. For Integrals

$$\text{Avg value} = \left[ \int_0^5 3 \cdot 2^{x-1} dx \right] \frac{1}{5}$$

$$= \left[ 3 \cdot 2^{x-1} \frac{1}{\ln 2} \Big|_0^5 \right] \frac{1}{5}$$

$$= \left[ 3 \cdot 2^4 \frac{1}{\ln 2} - 3 \cdot 2^{-1} \frac{1}{\ln 2} \right] \frac{1}{5}$$

$$3 \cdot 2^{c-1} = \left[ 3 \cdot 2^4 \frac{1}{\ln 2} - 3 \cdot 2^{-1} \frac{1}{\ln 2} \right] \frac{1}{5}$$

$$2^{c-1} = \frac{1}{5 \ln 2} (2^4 - 2^{-1})$$

~~$$c-1 = \log_2 \left( \frac{1}{5 \ln 2} (2^4 - 2^{-1}) \right) \Rightarrow \log_2 (2^4 - 2^{-1})$$~~

Solve by graphing...

$$c = 3.161$$

B