11-40 $\langle \frac{1}{25}t^3-50, \frac{1}{2}t^2+30 \rangle$

a) At $t=10$, the position vector is $\langle -10, 80 \rangle$, but this vector can have an initial point anywhere in the x-y plane. Vectors only show magnitude & direction.

b) If, at $t=0$, the marble's location is at (25, 12), then the actual position functions are:

$$x(t)=\frac{1}{25}t^3+25 \qquad y(t)=\frac{1}{2}t^2+12$$

@ $t=10$...

$$x(10)=65 \qquad y(10)=62$$

The position at $t=10$ is $\boxed{(65, 62)}$

c) position vectors don't account for initial position, only change in position. Therefore, they can't really be used to locate objects.

11-41 $x(t)=6t^2-0.4t^3 \quad y(t)=90+80\cos\left(\frac{\pi}{100}(6t^2-0.4t^3)\right)$

a) $t=0$. $y(0)=90+80\cos(\pi/100(0))=170$

$\boxed{170\text{ ft}}$

Is this the maximum height? Yes. If I wanted to, I'd maximize $y(t)$ to confirm this, but instead I will just look @ my graphing calculator, since I have it here.

b) $x'(t)=12t-0.4(3)t^2$. This is not a constant function, so the horizontal velocity will change.

c) velocity: $x'(t)=12t-1.2t^2$, $y'(t)=(-9.6\pi t+0.96\pi t^2)\sin\left(\frac{\pi}{100}(6t^2-0.4t^3)\right)$

@ $t=3$ $\langle 25.2, -61.895 \rangle$. Because $y'(t)<0$ at $t=3$, the coaster is descending.

→

11-41 d) $\|v(t)\| = \sqrt{(12t-1.2t^2)^2 + (-9.6\pi t + 0.96\pi t^2)^2 \sin^2(\frac{\pi}{100}(6t^2-0.4t^3))}$

$\|v(3)\| \approx 68.828$ ft/second, which is how fast the coaster will be moving when $t=3$.

11-42 a) $\langle t-\sin t, 1-\cos t\rangle$

$\frac{dy}{dt} = \sin t$ $\quad \frac{dx}{dt} = 1-\cos t$ $\quad \frac{dy}{dx} =$ ~~[illegible]~~ $\frac{\sin t}{1-\cos t}$

At $t = 2\pi/3$ seconds,

$\frac{dy/dt}{dx/dt} = \frac{dy}{dx}$

$\frac{dy}{dx} =$ ~~[illegible]~~ $\frac{\sqrt{3}/2}{1+1/2} = \boxed{\frac{\sqrt{3}}{3}}$

This is a positive slope ↔ so the gum is <u>rising</u> at this time

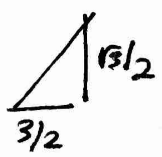
b) $\frac{dy}{dx}$ is <u>not</u> the velocity. velocity is change <u>in position</u> with respect to <u>time</u>.

$\frac{dy}{dx}$ is change in the vertical position w/ respect to the horizontal position.

c) $\vec{V}(t) = \langle 1-\cos t, \sin t\rangle$

$\vec{V}(\frac{2\pi}{3}) = \langle 3/2, \sqrt{3}/2\rangle$. It has two components b/c it's moving in both the x & y direction. i.e. 2D motion.

d) <u>speed</u> = hypotenuse of



$= \sqrt{\frac{3}{4}+\frac{9}{4}}$

$= \sqrt{\frac{12}{4}}$

$f\boxed{\sqrt{3}\text{ ft/sec}}$

11-43 $x(t) = 4\cos\left(\frac{\pi}{4} - t^3\right)$ $y(t) = 2\sin\left(\frac{\pi}{4} - t^3\right)$

a) $t = 0$

$$\left.\frac{dx}{dt}\right|_{t=0} = 4 \cdot (-3(0)^2)\cos\left(\frac{\pi}{4}\right) = \boxed{0}$$

$$\left.\frac{dy}{dt}\right|_{t=0} = 2(-3(0)^2)\sin\left(\frac{\pi}{4}\right) = \boxed{0}$$

b) "you sneak!" $\frac{dy/dt}{dx/dt} = \frac{0}{0}$ which is undefined.

c) $x = 4\cos(\pi/4 - t^3)$

$$\frac{x}{4} = \cos(\pi/4 - t^3)$$

$\pi/4 - t^3$ ... okay this method is a mess. I'll try this instead:

$$\frac{4\sin^2(\pi/4 - t^3)}{4} + \frac{16\cos^2(\pi/4 - t^3)}{16} = 1$$

$$\frac{y^2}{4} + \frac{x^2}{16} = 1$$

$$\frac{y^2}{4} = \frac{-x^2}{16} + 1$$

$$y^2 = -\frac{x^2}{4} + 4$$

$$y = \sqrt{4 - \frac{x^2}{4}}$$

At $t=0$,

$x = 4\cos(\pi/4) = 2\sqrt{2}$

$y = 2\sin(\pi/4) = \sqrt{2}$

Eliminate the parameter

$$\frac{dy}{dx} = \frac{1}{2}\left(4 - \frac{x^2}{4}\right)^{-1/2} \cdot \left(\frac{-2x}{4}\right)$$

$$\left.\frac{dy}{dx}\right|_{x=2\sqrt{2}} = \frac{1}{2}(4 - 2)^{-1/2} \cdot \left(-\frac{4\sqrt{2}}{4}\right)$$

$$= \frac{-\sqrt{2}}{2\sqrt{2}} = \boxed{-\frac{1}{2}}$$

d) slope $= -1/2$

$x(\theta) = 4\cos(\pi/4) = 2\sqrt{2}$

$y(0) = 2\sin(\pi/4) = \sqrt{2}$

Point-slope form:

$$\boxed{y-\sqrt{2} = -1/2(x-2\sqrt{2})}$$

e) if $\frac{dx}{dt} = 0$ & $\frac{dy}{dt} \neq 0 \Rightarrow \frac{dy}{dx}$ is undefined.

if $\frac{dy}{dt} = 0$ & $\frac{dx}{dt} \neq 0 \Rightarrow \frac{dy}{dx} = 0$

---

Review/preview

11-44 $y = xe^{x}$ $\frac{dx}{dt} = 4$ ← horizontal velocity

"How fast is distance from the origin changing when $x = -1$"

let $z$ = distance from the origin

I'm looking for $\frac{dz}{dt}$

$$x^2 + y^2 = z^2$$

$$x^2 + (xe^{x})^2 = z^2$$

$$x^2 + x^2e^{2x} = z^2$$

$$2x \cdot \frac{dx}{dt} + 2x\frac{dx}{dt}e^{2x} + x^2e^{2x} \cdot 2 \cdot \frac{dx}{dt} = 2z\frac{dz}{dt}$$

$x = -1, \frac{dx}{dt} = 4 \rightarrow$ $-2(4) + (-2)4e^{-2} + e^{-2}(2)(4) = 2z\frac{dz}{dt}$

$$-8 = 2z\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{-4}{z}$$

Find $z$ in terms of $x$:

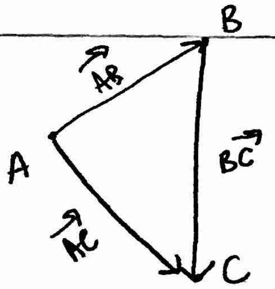
$z = \sqrt{x^2 + x^2e^{2x}}$

when $x = -1$

$z = \sqrt{1+e^{-2}}$

$$\frac{dz}{dt} = \boxed{\frac{-4}{\sqrt{1+e^{-2}}}}$$

11-45



$\vec{AB} + \vec{BC} = \vec{AC}$

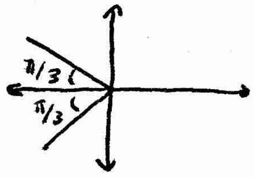
$$\boxed{\vec{BC} = \vec{AC} - \vec{AB}}$$

11-46 $r = 3 + 6\cos(\theta)$

When does it cross the pole?

$0 = 3 + 6\cos\theta$

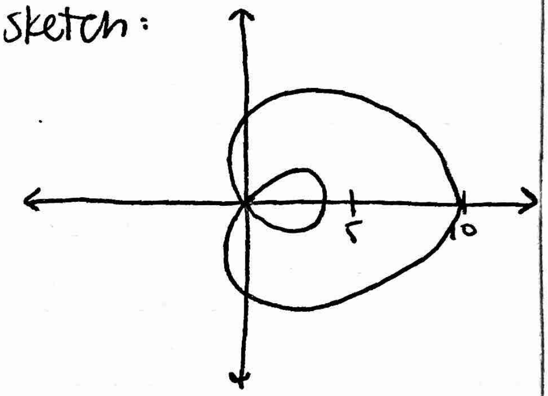
$\cos\theta = -1/2$



$\theta = \pm\frac{2\pi}{3} + 2\pi k$

$\theta = \{\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}, \ldots\}$

sketch:



$$\int_{2\pi/3}^{4\pi/3} \frac{1}{2}(3+6\cos\theta)^2 d\theta \approx 4.892$$ ← inner loop

$$\int_{4\pi/3}^{8\pi/3} \frac{1}{2}(3+6\cos\theta)^2 d\theta \approx 79.931$$ ← entire limaçon (outer loop)

$$\frac{4.892}{79.931} \approx \boxed{6.120\%}$$

11-47 $g(h(t)) = \cos\sqrt{t}$, $g(t) = \cos(t)$, $h(t) = \sqrt{t}$

a) $\frac{dg}{dt} = -\sin(t)$ $\frac{dh}{dt} = \frac{1}{2}t^{-1/2} = \frac{1}{2\sqrt{t}}$

b) $$\frac{\frac{dg}{dt}}{\frac{dh}{dt}} = \frac{dg}{dh}$$

$$\frac{-\sin t}{1/2\sqrt{t}} = -\sin[h(t)^2] \cdot 2h(t)$$ ← since $g(t) = \cos(t)$ $= \cos(h(t))^2$

$-2\sqrt{t}\sin t = -2\sqrt{t}\sin t$ ✓

11-48 $\langle \underset{\substack{\uparrow \\ \text{x position}}}{x(t)}, \underset{\substack{\uparrow \\ \text{y position}}}{y(t)} \rangle \rightarrow \langle \underset{\substack{\uparrow \\ \text{x velocity}}}{x'(t)}, \underset{\substack{\uparrow \\ \text{y velocity}}}{y'(t)} \rangle = \underset{\text{same}}{\langle \frac{dx}{dt}, \frac{dy}{dt} \rangle}$

$\boxed{C}$ both of the above ↑ →

11-49 $y = x^{3/2}$ from $x=0$ to $x=4$ arc length

$$\int_0^4 \sqrt{1+(\tfrac{3}{2}x^{1/2})^2}\,dx = \int_0^4 \sqrt{1+\tfrac{9}{4}x}\,dx \approx 9.073 \quad \boxed{E}$$

11-50 $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$

$$\rho = \lim_{n\to\infty} \left| \frac{x^{n+1}}{n+1} \cdot \frac{n}{x^n} \right| = \lim_{n\to\infty} \left| \frac{xn}{n+1} \right| = |x| \qquad -1<x<1$$

Check $x=-1$: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(-1)^n}{n} = \sum_{n=1}^{\infty} \frac{-1}{n}$ diverges (harmonic)

Check $x=1$: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ converges (alt. harmonic)

$(-1,1]$ $\boxed{D}$

11-51 Area under $f(x)=2x^2-3x+1$ over $1 \le x \le 5$. 4 Righthand rectangles.

$\frac{5-1}{4} = 1$ ← base of each rectangle $\Delta x$

Area $\approx 1f(2)+1f(3)+1f(4)+1f(5)$

$2(4+9+16+25) - 3(2+3+4+5) + 1+1+1+1$

$= 2(54) - 3(14) + 4$

$= 108 - 42 + 4$

$= 112 - 42$

$= 70$ $\boxed{E}$

11-52 $x=\sqrt{4t^2+1}$ $y=2t$ $t>0$

$t=y/2$

$x=\sqrt{4(\frac{y}{2})^2+1}$

$x=\sqrt{y^2+1}$ $\boxed{A}$

11.2.2

11-53 $x(t)=e^{2t}$ $y(t)=t^2+3$

a) velocity: $x'(t)=2e^{2t}$ , $y'(t)=2t$

b) velocity: $\langle 2e^{2t}, 2t \rangle$

c) speed = $\|\vec{v}(t)\|$

$= \sqrt{(2e^{2t})^2+(2t)^2}$

d) slope of curve, $\frac{dy}{dx}$ at time t

$\frac{dy}{dt}=2t$ $\frac{dx}{dt}=2e^{2t}$ $\boxed{\frac{dy}{dx}=\frac{2t}{2e^{2t}}=\frac{t}{e^{2t}}}$

e) $\frac{dy}{dx}$ does not represent velocity. velocity is Δ position w/ respect to time. $\frac{dy}{dx}$ is change in y-value w/respect to x-value.

11-54 $x(t)=99t^{1/3}$ $y(t)=20t^{1/2}$ $t \geq 0$

a) $\vec{a}(t)$ ...

$\vec{v}(t)=\langle \frac{1}{3}(99)t^{-2/3}, 10t^{-1/2} \rangle$

$\vec{a}(t)=\langle -22t^{-5/3}, -5t^{-3/2} \rangle$

$\vec{a}(64\rangle = \langle -22(4)^{-5}, -5(8)^{-3} \rangle = \langle \frac{-22}{1024}, \frac{-5}{512} \rangle = \boxed{\langle \frac{-11}{512}, \frac{-5}{512} \rangle}$

The chalk has negative acceleration in both directions, which means that the velocity is decreasing in both directions

ⓑ $\frac{\frac{d^2y}{dt^2}}{\frac{d^2x}{dt^2}} = \frac{d^2y}{d^2x}$ ← This is taking the second derivative w/ respect to t of x & y and dividing them, which will not give you $\frac{d^2y}{dx^2}$ ← The derivative w/ respect to x

11-54 c) $x(t) = 99t^{1/3} \rightarrow t^{1/3} = \frac{x}{99}$

$t = \left(\frac{x}{99}\right)^3$

$y = 20t^{3/2}$

$y(x) = 20\left(\frac{x}{99}\right)^{3/2}$ ← eliminate parameter

$\frac{dy}{dx} = \frac{20}{99}\left(\frac{3}{2}\right)\left(\frac{x}{99}\right)^{1/2}$

$\frac{d^2y}{dx^2} = \frac{30}{99}\left(\frac{1}{2}\right)\left(\frac{x}{99}\right)^{-1/2}$

when $t = 64$, $x = 99(4) = 396$

$\left.\frac{d^2y}{dx^2}\right|_{x=396} = \frac{30}{198}\left(\frac{396}{99}\right)^{-1/2} \approx \underline{0.152 > 0}$

concave up!

d) NO! It represents the concavity of the <u>path</u> of the particle. acceleration is w/respect to <u>time</u>. This is w/ respect to <u>x</u>

11-55 $x(t)$ $y(t)$ $t \geq 0$

when $t = 0$, $x = y = 0$

$x'(t) = 2^{\sqrt{t}}$ $\frac{dy}{dx} = t^2$

a) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$t^2 = \frac{y'(t)}{2^{\sqrt{t}}}$

$\boxed{y'(t) = 2^{\sqrt{t}}t^2}$

b) acceleration: $\langle 2^{\sqrt{t}}\ln 2 \cdot \frac{1}{2}t^{-1/2}, 2^{\sqrt{t}}\ln 2 \cdot \frac{1}{2}t^{-1/2}t^2 + 2t\,2^{\sqrt{t}} \rangle$

at $t=1$ ... $\langle (2)(\ln 2)\left(\frac{1}{2}\right), 2\ln 2 \cdot \frac{1}{2} + 2 \cdot 2 \rangle$

$\boxed{\langle \ln 2, \ln 2 + 4 \rangle}$

→

c) concavity: $\frac{d^2y}{dx^2}$

$\frac{dy}{dx} = \frac{2^{\sqrt{t}}t^2}{2^{\sqrt{t}}} = t^2$ $\frac{d}{dt}\left(\frac{dy}{dx}\right) = 2t$

$\frac{dx}{dt} = 2^{\sqrt{t}}$

$\frac{d^2y}{dx^2} = \frac{2t}{2^{\sqrt{t}}}$

@ t=1 ... concavity $= \frac{2(1)}{2^1} = 1$. concave <u>up</u>

———— REVIEW/PREVIEW ————

11-56 Catch the ball @ t=6. Throw the ball @ t=3.

$x(t) = 40t - 120$
$y(t) = -16t^2 + 144t - 282$

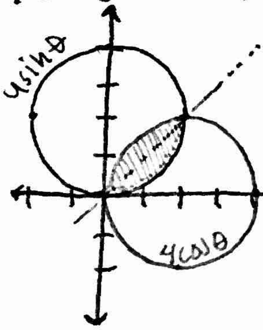
When t = 6, Pete Moss catches it @ $y(6) = \underline{\underline{6}}$
6 ft above ground

a) $x'(t) = 40$
$y'(t) = -32t + 144$ $\vec{v}(t) = \langle 40, -32t + 144 \rangle$

$\vec{a}(t) = \langle 0, -32 \rangle$

b) @ t=6 ...
$\vec{v}(6) = \langle 40, -32(6) + 144 \rangle = \langle 40, -48 \rangle$
$\vec{a}(6) = \langle 0, -32 \rangle$

11-57 $r = 4\cos\theta$ $r = 4\sin\theta$ ← both circles



$4\cos\theta = 4\sin\theta$
intersect @
$\theta = \pi/4$

→ split into 2 integrals

$4\cos\theta = 0 \rightarrow \theta = \frac{\pi}{2} + \pi k$

$\int_{\pi/4}^{\pi/2} \frac{1}{2}(4\cos\theta)^2 d\theta$

.... see next page

$4\sin\theta = 0 \rightarrow \theta = \pi k$

$\int_{\pi/4}^{\pi/2} \frac{1}{2}(4\cos\theta)^2 d\theta \quad + \quad \int_{0}^{\pi/4} \frac{1}{2}(4\sin\theta)^2 d\theta$

$= 8\int_{\pi/4}^{\pi/2} \cos^2\theta \, d\theta \quad + \quad 8\int_{0}^{\pi/4} \sin^2\theta \, d\theta$

$= 4\int_{\pi/4}^{\pi/2} (1+\cos 2\theta) d\theta \quad + \quad 4\int_{0}^{\pi/4} (1-\cos 2\theta) d\theta$

$= \left[4\theta + 2\sin 2\theta\right]_{\pi/4}^{\pi/2} \quad + \quad \left[4\theta - 2\sin 2\theta\right]_{0}^{\pi/4}$

$= 2\pi - (\pi + 2) \quad + \quad (\pi - 2) - (0 - 0)$

$= \boxed{2\pi - 4 \text{ units}^2}$

11-58 $y = e^x \cos(x)$ on $[-1, 5]$

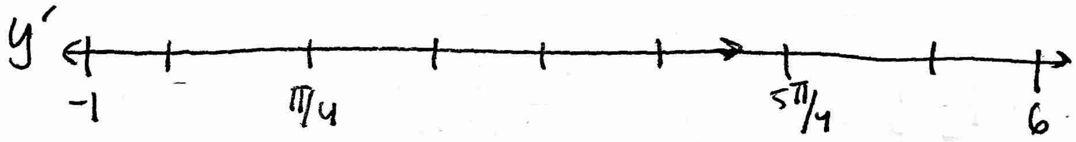
$y' = -e^x \sin x + e^x \cos x = e^x(\cos x - \sin x)$

$y' = 0 \rightarrow \quad e^x = 0 \quad \cos x - \sin x = 0$

never

$\cos x = \sin x$

$x = \pi/4 + \pi k$



$y'$ + — +

inc dec inc

Increasing on $(-1, \pi/4) \cup (5\pi/4, 6)$ b/c $y'(x) > 0$

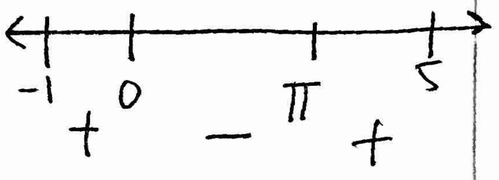
Decreasing on $(\pi/4, 5\pi/4)$ b/c $y'(x) < 0$

$y'' = e^x(\cos x - \sin x) + e^x(-\sin x - \cos x)$

$y'' = e^x(-2\sin x)$

$y'' = 0 \rightarrow e^x = 0$ never $\quad -2\sin x = 0$

$x = \pi k$



concave up: $(-1, 0) \cup (\pi, 5)$ b/c $y''(x) > 0$ $y''$

concave down: $(0, \pi)$ b/c $y''(x) < 0$

→

11-58 continued...

**Points of inflection** @ $x=0$ and $x=\pi$ because $y''$ changes sign.

$y(0)=1$ $\qquad$ $\underline{(0,1) \quad (\pi,5)}$

$y(\pi)=5$

**Intercepts:** $y=e^x\cos x$ $\qquad$ $\underline{x\text{-Intercepts at}}$

$0=e^x\cos x$ $\qquad$ $(\pi/2,0)\ (3\pi/2,0)$

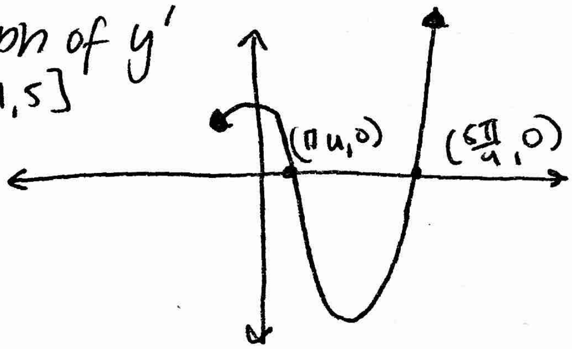
$e^x \neq 0$ never $\qquad \cos x=0$ $\qquad$ because $y=0$

$x=\pi/2+\pi k$

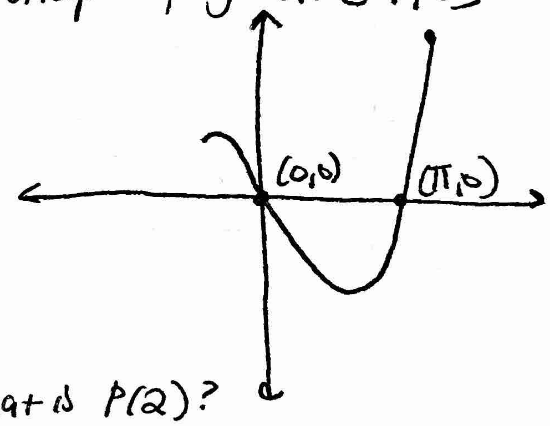
$y\text{-int}=e^0\cos 0$

$\underline{y\text{-int}=1 \quad b/c \quad x=1}$

Graph of $y'$ on $[-1,5]$



Graph of $y''$ on $[-1,5]$



11-59 $\frac{dP}{dt}=0.01P(100-P)$ $\quad P(0)=50$ $\quad$ what is $P(2)$?

$$\int\frac{dP}{P(100-P)}=\int 0.01\,dt$$

partial fractions:

$1=A(100-P)+BP$

$P=100:\ 1=100B$ $\qquad$ $P=0:\ 1=100A$

$1/100=B$ $\qquad$ $1/100=A$

$$\int 0.01\,dt=\frac{1}{100}\int\frac{dP}{P}+\frac{1}{100}\int\frac{dP}{100-P}$$

$$0.01t=\frac{1}{100}\ln\left|\frac{P}{100-P}\right|$$

when $t=2$...

$$0.02=\frac{1}{100}\ln\left|\frac{P}{100-P}\right|$$

$$2=\ln\left|\frac{P}{100-P}\right|$$

$$\frac{P}{100-P}=e^2$$

$$P=e^2(100-P)$$

$$P=100e^2-e^2P$$

$$P(1+e^2)=100e^2$$

$$P=\frac{100e^2}{1+e^2}\doteq\boxed{88\text{ fish}}$$

11-60 $\sum_{n=1}^{\infty} \frac{x^{2n+1}}{n!}$

$\lim_{n\to\infty}\left|\frac{x^{2n+3}}{(n+1)!}\cdot\frac{n!}{x^{2n+1}}\right|$

$=\lim_{n\to\infty}\left|\frac{x^{2}}{n+1}\right| = 0<1$ converges for all x $(-\infty,\infty)$

11-61 $f(x)=3x^{1/3}-4x$ $x_1=1$ SKIP "NEWTON'S METHOD"

# 11.2.3

11-62 $\vec{v}(t) = \left\langle \frac{1}{2}e^{t/2}, \sqrt{2t+1} \right\rangle$

t : seconds
$\vec{v}(t)$ : ft/sec

starting position is (0,1)

a) velocity = $\frac{d \text{ Position}}{d \text{ time}}$ $\Rightarrow$ position = $\int$ velocity dt

$$\vec{p}(t) = \left\langle \int \frac{1}{2}e^{t/2}dt, \int \sqrt{2t+1}\,dt \right\rangle \rightarrow$$

$$= \left\langle e^{t/2} + C, \frac{1}{2}\cdot\frac{2}{3}(2t+1)^{3/2} + C \right\rangle$$

$$p(0) = \langle 0, 1 \rangle$$

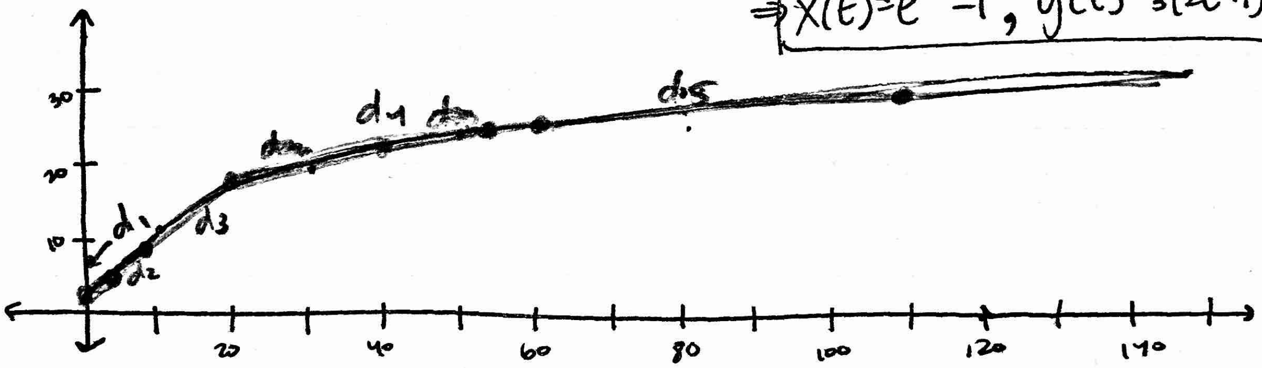
- $e^0 + C = 0$ $\quad$ $\frac{1}{3}(1)^{3/2} + C = 1$

$C = -1$ $\quad$ $C = 2/3$

$$\vec{p}(t) = \left\langle e^{t/2} - 1, \frac{1}{3}(2t+1)^{3/2} + \frac{2}{3} \right\rangle$$

BUT parametric gives actual position

$$\Rightarrow \boxed{x(t) = e^{t/2} - 1, \; y(t) = \frac{1}{3}(2t+1)^{3/2} + \frac{2}{3}}$$



| time | x | y | ΔX | Δy | ΔZ ← distance |
|---|---|---|---|---|---|
| 0 | 0 | 1 | | | |
| 2 | 1.718 | 4.393 | 1.718 ≈ 2 | ≈ 3 | $\sqrt{4+9} \approx 3.6$ |
| 4 | 6.389 | 9.667 | 4.6[illegible] ≈ 5 | ≈ 5 | $\sqrt{25+25} = 5$ |
| 6 | 19.086 | 16.291 | 12.697 ≈ 13 | ≈ 6 | $\sqrt{13^2+6^2} \approx 14.3$ |
| 8 | 53.598 | 24.031 | 34.512 ≈ 35 | ≈ 8 | $\sqrt{35^2+8^2} \approx 36$ |
| 10 | 147.413 | 32.745 | 93.815 ≈ 94 | ≈ 8 | $\sqrt{94^2+8^2} \approx 94$ |

Total ≈ 152.9

It 63 arclength on $[a,b] = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

$x(t) = 8t - 2t^2,\ y(t) = 0$

a) This is the same as $x(t) = 8t - 2t^2$ b/c there is no change in vertical position.

b) $0 \leq t \leq 3$

$x(0) = 0 \quad x(3) = 24 - 2(9) = 6$

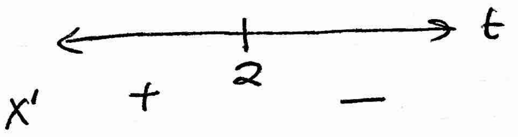
It starts @ $(0,0)$ and at $t=3$, it is at $(6,0)$

$x'(t) = 8 - 4t$

$0 = 8 - 4t$

$4t = 8$

$t = 2$



$x(2) = 16 - 2(4)$

$x(2) = 8$

It moves to the right until $t=2$, when it reaches $(8,0)$. Then it changes direction and moves toward $(6,0)$.

c) Displacement = change in position = $\int_0^3 v(t)\,dt$

$\int_0^3 (8-4t)\,dt = 8t - 2t^2\Big|_0^3 = 24 - 2(9) = \underline{6 \text{ units}}$ ← displacement

Distance traveled - predict: 10 (8+2)

$\int_0^3 |8-4t|\,dt = \int_0^2 (8-4t)\,dt - \int_2^3 (8-4t)\,dt$

$= 8(2) - 2(2)^2 - 0 - \left[8(3) - 2(3)^2 - 8(2) + 2(2)^2\right]$

~~[illegible]~~ 800

~~[illegible]~~

$= 16 - 8 - (24 - 18 - 16 + 8)$

$= 32 - 16 - 24 + 18$

$= \boxed{10}$ units ← this is the arclength

11-63 d) speed:

$$\text{Position} = p(t) = 8t - 2t^2$$

$$\text{velocity} = \boxed{v(t) = 8-4t}$$

~~speed~~

$$\text{As a vector: } \boxed{\langle 8-4t, 0\rangle = \vec{v}(t)}$$

e) $\text{Speed} = |v(t)| = |8-4t|$

$$\text{speed vector form} = \|\vec{v}(t)\| = \sqrt{(8-4t)^2+0^2} = \sqrt{(8-4t)^2}$$

f) $\text{Distance}: \int_0^3 |8-4t|\,dt$ $\qquad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} =$

$\int |v(t)|\,dt$

$$\text{Distance using vector} = \int_0^3 \sqrt{(8-4t)^2+0^2}\,dt$$

11-64 $x(t)=\cos t$ $\quad y(t)=\sin t$ on $0 \le t \le 2\pi$ creates a circle.

a) i) $\int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2+\left(\frac{dx}{dt}\right)^2}\,dt$

$= \int_0^{2\pi} \sqrt{(\cos^2 t)+(\sin^2 t)}\,dt$

$= \int_0^{2\pi} dt$

$= \boxed{2\pi}$ ← This is the circumference of a unit circle!

ii) $\int_0^{4\pi} \sqrt{\cos^2 t + \sin^2 t}\,dt$

$= \int_0^{4\pi} dt$

$= \boxed{4\pi}$ This is twice as big b/c you go around the circle TWICE!

11-64 b) $x(t)=3t-2,\ y(t)=4t-1$ on $2\leq t\leq 5$

$x+2=3t$

$\frac{x+2}{3}=t$

$y=4\left(\frac{x+2}{3}\right)-1$

$y=\frac{4}{3}x+\frac{8}{3}-\frac{3}{3}$

$y=\frac{4}{3}x+\frac{5}{3}$

if $2\leq t\leq 5$

$6\leq 3t\leq 15$

$4\leq 3t-2\leq 13$

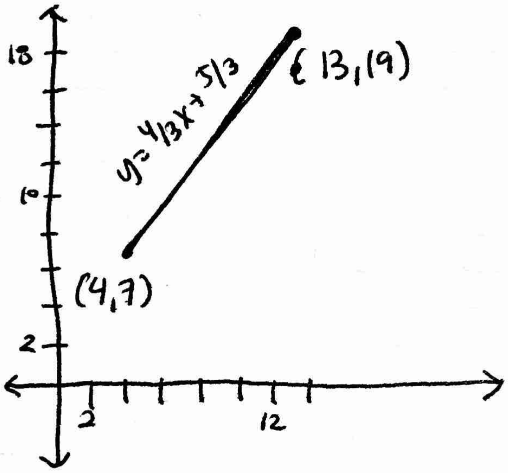
$4\leq x\leq 13$

$2\leq t\leq 5$

$8\leq 4t\leq 20$

$7\leq 4t-1\leq 19$

$7\leq y\leq 19$



Method 1 - distance formula

$d=\sqrt{(13-4)^2+(19-7)^2}$

$=\sqrt{9^2+12^2}$

$=\sqrt{81+144}$

$=\sqrt{225}$

$=\boxed{15}$

Method 2:

Use parametric arc length! okay.

$$\int_2^5\sqrt{(3)^2+(4)^2}\ dt$$

$\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2$

$=\int_2^5 5\,dt$

$=5t\Big|_2^5$

$=25-10$

$=\boxed{15}$

11-64 c) well... the particle is traveling <u>along</u> the <u>curve</u>.

11-65 $x(t)=t-\sin(t)$ $y(t)=1-\cos(t)$

"one cycle" is one period... so $2\pi$

$0\leq t\leq 2\pi$

length: $\int_0^{2\pi}\sqrt{(1-\cos t)^2+(0+\sin t)^2}\;dt$

$=\int_0^{2\pi}\sqrt{1-2\cos t+\cos^2 t+\sin^2 t}\;dt$

$=\int_0^{2\pi}\sqrt{2-2\cos t}\;dt$

... calculator...

$=\boxed{8}$

11-66 $x=\sin(7t)$ $y=\cos(5t)$ is a Lissajous curve. (The basket-looking thing.

$\int_0^{2\pi}\sqrt{(7\cos(7\theta))^2+(-5\sin(5t))^2}\,dt \approx 36.468$ units.

REVIEW + PREVIEW

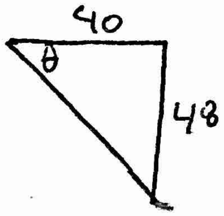
11-67 $x(t)=40t-120$, $y t)=-16t^2+144t-282$ $\underset{\substack{\uparrow\\ \text{seconds}}}{y(6)}=\underset{\substack{\uparrow\\ \text{ft above ground}}}{6}$

a) when $t=6$, $\vec{v}(6)=\langle 40,-16(2)(6)+144\rangle$

$\boxed{\vec{v}(6)=\langle 40,-48\rangle}$

speed $=\|\vec{v}\|=\sqrt{40^2+48^2}\approx \underline{62.482 \text{ ft/sec}}$

b)

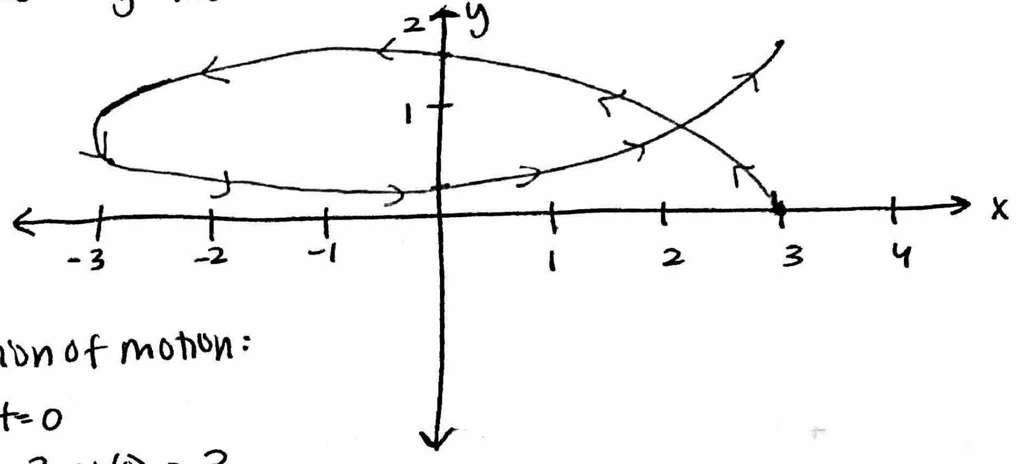


$\theta=\arctan\left(\frac{48}{40}\right)\approx 0.876=\underline{50.194°}$ from the horizontal

→

11-68 $x = 3\cos(2t)$ $y = \ln(1+t) + \sin(2t)$ ← position on $0 \le t \le \pi$

a)



Direction of motion:

time $t=0$

$x(0) = 3\cos(0) = 3$

$y(0) = \ln 1 + \sin 0 = 0$

moves up and ~~to~~ the left from this point

b) maximum $y$: $y' = \frac{1}{1+t} + 2\cos(2t) = 0$

$$2\cos(2t) = \frac{-1}{1+t}$$

sad ^^

oh it says use your graphing calculator

Graph $y'$ and find zeros!

$y'$ changes from positive to negative @ $t = 0.917$

$y(0.917) = \ln(1.917) + \sin[2(0.917)] = 1.616$

$x(0.917) = 3\cos[2(0.917)] = -0.781$

when $y$ is @ max value, the position of the object is $\underline{(-0.781, 1.616)}$

d) $\vec{a}(t)$: $\vec{v}(t) = \langle -2 \cdot 3\sin(2t), \frac{1}{1+t} + 2\cos(2t) \rangle$

$$\boxed{\vec{a}(t) = \langle -12\cos(2t), \frac{-1}{(1+t)^2} - 4\sin(2t) \rangle}$$

→

11-69 a) $\int \frac{x^2}{x^2+1}\,dx$   parts: $u = x^2$   $du = 2x\,dx$

$v = \arctan x$   $dv = \frac{1}{x^2+1}\,dx$

= ... wait that is a bad idea. $\int v\,du$ is ugly.

Trig substitution?

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\tan^2 x + 1 = \sec^2 x$$

so let $x = \tan\theta$   $dx = \sec^2\theta\,d\theta$

$$\int \frac{\tan^2\theta}{\tan^2\theta + 1} \cdot \sec^2\theta\,d\theta$$

$$= \int \frac{\tan^2\theta\sec^2\theta}{\sec^2\theta}\,d\theta$$

$$= \int \tan^2\theta\,d\theta$$

$$= \int (\sec^2\theta - 1)\,d\theta$$

$= \tan\theta - \theta$ , but $\theta = \arctan x$

$$= \tan(\arctan x) - \arctan x + C$$

$= \underline{x - \arctan x + c}$   That was excessive

b) $\int_1^\infty \frac{1}{e^x}\,dx$

$$= \lim_{b\to\infty} \int_1^b e^{-x}\,dx$$

$$= \lim_{b\to\infty} \left[ -e^{-x} \Big|_1^b \right]$$

$$= \lim_{b\to\infty} \left[ -e^{-b} + e^{-1} \right]$$

$$= \boxed{e^{-1}}$$

11-69 c) $\int \sec(2x)\tan(2x)\,dx$ why the trig though

$\uparrow$ $d/dx \sec(2x)$

$= \frac{1}{2}\sec(2x) + C$ phew

11-70 a) $\sum_{n=1}^{\infty} \frac{(-1000)^n}{n!} = \sum_{n=1}^{\infty} (-1)^n \frac{1000^n}{n!}$ $\left\{\frac{1000^n}{n!}\right\}$ is positive & decreasing

$\lim_{n\to\infty} \frac{1000^n}{n!} = 0$

$\therefore$ by AST the series converges.

w/absolute value... $\sum_{n=1}^{\infty} \frac{1000^n}{n!}$

ratio test $\lim_{n\to\infty} \left| \frac{1000^{n+1}}{(n+1)!} \cdot \frac{n!}{1000^n} \right|$

$= \lim_{n\to\infty} \left| \frac{1000}{n+1} \right|$

$= 0$ $\therefore$ converges absolutely

The answer key says "conditionally" though...
But wolfram Alpha agrees with me...

b) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n}}$ $\lim_{n\to\infty} \frac{n}{\sqrt{n}} \neq 0$ $\therefore$ Diverges by $n^{th}$ term test

c) $\sum_{n=1}^{\infty} \frac{6}{n^{5/2}} = 6 \sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$ converges absolutely by p-series

d) $\sum_{n=1}^{\infty} e^{-n}$ is geometric with $|r| = \frac{1}{e} < 1$

$\therefore$ it converges absolutely.

11-71 $\frac{d}{dx}\int_0^{x} f'(t)\,dt = f'(x)$

$f'(6) = 0$ $\boxed{A}$

11-72 $R' = 5 - \frac{1}{2}(t-3)^2$ R = revenue . t = time (months)

max R:

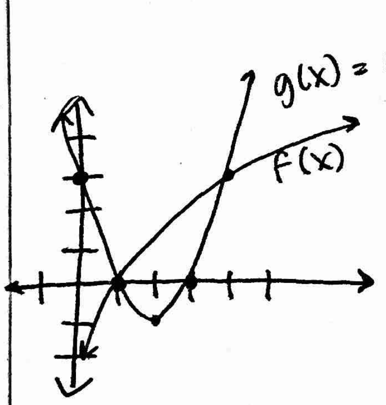
$0 = 5 - \frac{1}{2}(t-3)^2$

$(t-3)^2 = 10$

$t = \sqrt{10} + 3 \approx \boxed{6 \text{ months}}$

11-73 The slope field matches $\boxed{C}$ because all of the slopes are opposite the y-values. they don't depend on x

11-74



rotate about y-axis.

outer radius = g(x)

inner radius = f(x)

~~xogo~~ y goes from -1 to 3

... This problem is using the "shell" method

... SKIP IT

11-75 $\boxed{B}$ $f(2) < f''(2) < f'(2)$

$\uparrow$ negative $\uparrow$ looks like 0 $\uparrow$ positive