11-15 $r(\theta) = 7 + 6\cos\theta$ 0

a) $0 \leq \theta \leq 2\pi$ (1 period)

b) $\int_0^{2\pi} \frac{1}{2}(7+6\cos\theta)^2 d\theta$ $\cancel{= \frac{7}{2}\int_0^{2\pi} d\theta + 3\int_0^{2\pi} \cos}$

$= \int_0^{2\pi} \frac{1}{2}(49 + 84\cos\theta + 36\cos^2\theta)\,d\theta$

$= \int_0^{2\pi} \frac{49}{2} d\theta + \int_0^{2\pi} 42\cos\theta\, d\theta + \int_0^{2\pi} 18\cos^2\theta\, d\theta$ ← from HW problem 11-6 $= 18\pi$

$= \frac{49}{2}\theta\Big|_0^{2\pi} + 42\sin\theta\Big|_0^{2\pi} + 18\pi$

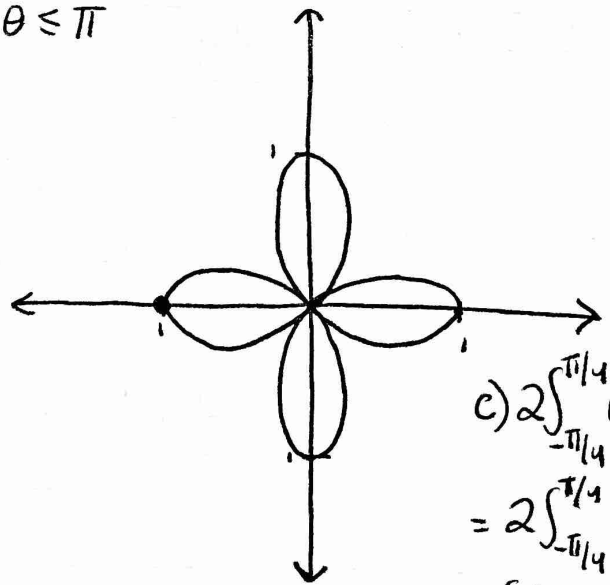
$= 49\pi + 18\pi$

$= \boxed{67\pi} \approx 210.487$ units^2

c) my answer was 210.487 also!

11-16 $r = \cos(2\theta)$ on $-\pi \leq \theta \leq \pi$

θ	r
$-\pi$	1
$-3\pi/4$	0
$-\pi/2$	-1
$-\pi/4$	0
0	1
$\pi/4$	0
$\pi/2$	-1
$3\pi/4$	0
π	1



c) $2\int_{-\pi/4}^{\pi/4} \cos^2(2\theta)\, d\theta$

$= 2\int_{-\pi/4}^{\pi/4} \frac{1}{2}[\cos(4\theta)+1]\, d\theta$

$= \left[\frac{1}{4}\sin(4\theta) + \theta\right]_{-\pi/4}^{\pi/4}$

$= [0 + \pi/4] - [0 - \pi/4]$

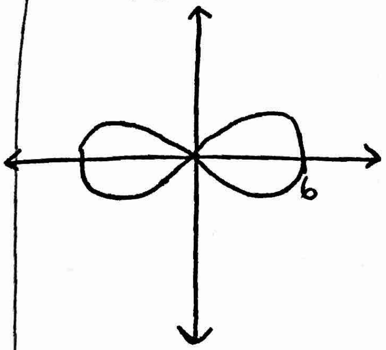
$= \boxed{\frac{\pi}{2}}$

a) $\theta = \left\{\pm\frac{3\pi}{4}, \pm\frac{\pi}{4}\right\}$

b) $\underbrace{4\int_{-\pi/4}^{\pi/4} \frac{1}{2}\cos^2(2\theta)\, d\theta}_{\text{one of the "leaves"}}$

4 leaves = entire area

11-17 $r = \sqrt{36\cos 2\theta}$



Plan:

Step 1 - Find the area of 1 leaf.

- figure out when the graph crosses the pole, i.e. when $r=0$

$$0 = \sqrt{36\cos 2\theta}$$
$$0 = 36\cos 2\theta$$
$$0 = \cos 2\theta$$
$$2\theta = \frac{\pi}{2} + \pi k$$
$$\theta = \frac{\pi}{4} + \frac{\pi}{2}k$$

- The graph will be @ pole at $\theta = \pi/4$, then cross again at $\theta = \frac{3\pi}{4}$

- Evaluate integral from $\pi/4$ to $3\pi/4$

$$\text{Area} = \int_a^b \frac{1}{2}[f(\theta)]^2 d\theta$$
$$= \int_{\pi/4}^{3\pi/4} \frac{1}{2}\left(\sqrt{36\cos 2\theta}\right)^2 d\theta$$
$$= 18\int_{\pi/4}^{3\pi/4} \cos(2\theta)\, d\theta$$
$$= 9\sin(2\theta)\Big|_{\pi/4}^{3\pi/4}$$
$$= -9-9$$
$$= -18$$

← actual area is $\underline{18}$ but it looks negative b/c of the direction of the curve

Step 2 - DOUBLE IT!

$$18 \cdot 2 = \boxed{36}$$

11-18 a) $r=\cos(2\theta)$ $\quad r=1$

for $r=1$... $\int_0^{2\pi}\frac{1}{2}R^2_{d\theta}=\int_0^{2\pi}1^2 d\theta = \pi$

for $r=\cos(2\theta)$ $\quad \cos(2\theta)=0$

$2\theta = \pi/2 + \pi k$

One leaf... $\quad \theta = \pi/4 + \pi/2 k$

$\int_{-\pi/4}^{\pi/4}\frac{1}{2}\cos^2(2\theta)d\theta$ $\quad \cos^2(2\theta)=\frac{1}{2}(\cos 4\theta + 1)$

$=\int_{-\pi/4}^{\pi/4}\frac{1}{4}(\cos 4\theta + 1)d\theta$

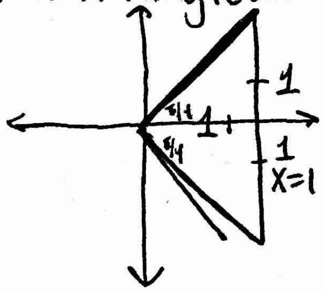
$=\frac{1}{16}(\sin(4\theta)+4\theta)\Big|_{-\pi/4}^{\pi/4} = \frac{1}{16}\left[(0+4\pi/4)-(0-4\pi/4)\right]=\frac{4\pi}{32}=\frac{\pi}{8}$

Four leaves...

$4\left(\frac{4\pi}{32}\right)=\frac{\pi}{2}$ ← This says $\boxed{\frac{\pi}{2}}$

b) $r=\sec\theta$, $\theta=\pi/4$, $\theta=-\pi/4$

Use a triangle...



Area $=\frac{1}{2}(2)1 = \boxed{1}$

NOTICE: The form $r=a\sin(n\theta)$ and $r=a\cos(n\theta)$ creates a "rose" with petals

when n is even, there are 2n petals

when n is odd, there are n petals.

11-19 $r=3\sin(3\theta)$ "rose curve" with 3 petals

Where do the petals start and end?

$0=3\sin(3\theta)$

$3\theta=\pi k$

$\theta=\frac{\pi}{3}k$

one petal/leaf: go from $0 \leq \theta \leq \pi/3$

$$\int_0^{\pi/3} \frac{1}{2}\left(3\sin(3\theta)\right)^2 d\theta$$

$$= \frac{9}{4}\int_0^{\pi/3} (1-\cos 6\theta)\, d\theta$$

$$= \frac{9}{4}\left[\theta - \frac{1}{6}\sin 6\theta\right]\Big|_0^{\pi/3}$$

$$= \frac{9}{4}\left[\frac{\pi}{3}\right] = \frac{3\pi}{4} \leftarrow \text{one petal}$$

Three petals $\boxed{9\pi/4}$

{Review/Preview}

11-20 a) $\sum_{n=1}^{\infty} \frac{-1}{n+2}$ compare to harmonic... it diverges by DCT

b) $\sum_{n=1}^{\infty} \ln n$ diverges by n^{th} term test

c) $\sum_{n=1}^{\infty} 0.1$ also diverges by n^{th} term test

d) $\sum_{n=1}^{\infty} \frac{1}{n^{1.01}}$ conv. by p-series test

11-21 $\vec{a} = 3\vec{i} + 5\vec{j}$ $\quad \vec{b} = -9\vec{j}$ $\quad \vec{c} = 6\vec{i} - 8\vec{j}$

a) $6\vec{a} + \vec{b} = 6(3\vec{i} + 5\vec{j}) + (-9\vec{j})$

$= 18\vec{i} + 30\vec{j} - 9\vec{j}$

$= \boxed{18\vec{i} + 21\vec{j} \quad \langle 18, 21 \rangle}$

b) $\frac{1}{2}\vec{c} + \vec{a} = 3\vec{i} - 4\vec{j} + 3\vec{i} + 5\vec{j}$

$= \boxed{6\vec{i} + \vec{j} \quad \langle 6, 1 \rangle}$

c) $\dfrac{5\vec{c}}{\|\vec{c}\|} = \dfrac{5(6\vec{i} - 8\vec{j})}{\sqrt{36+64}} = \dfrac{30\vec{i} - 40\vec{j}}{10} = \boxed{3\vec{i} - 4\vec{j} \quad \langle 3, -4 \rangle}$

d) $\|\vec{a} - \vec{b}\| = \|3i + 14j\| = \sqrt{9 + 196} = \boxed{\sqrt{205}}$

11-22 a) $\pi \int \frac{1}{2} \sec^2 x \tan x \, dx$

$u = \tan x \quad du = \sec^2 x \, dx$

$= \frac{\pi}{2} \int u \, du$

$= \frac{\pi}{2} \cdot \frac{u^2}{2} + C$

$= \boxed{\dfrac{\pi \tan^2 x}{4} + C}$

b) ~~$\frac{1}{2}$ tan sin~~

$\int \dfrac{\cos^2 x - \sin^2 x}{\sin 2x} \, dx$

$= \int \dfrac{\cos(2x)}{\sin(2x)} \, dx$

$u = \sin(2x) \quad \frac{1}{2} du = \cos(2x) \, dx$

$= \frac{1}{2} \int \frac{1}{u} \, du$

$= \frac{1}{2} \ln|u| + C$

$= \boxed{\frac{1}{2} \ln|\sin(2x)| + C}$

c) $\int_0^a (ax^{2/3} - b) \, dx$

$= \dfrac{3a x^{5/3}}{5} - bx \Big|_0^a$

$= \boxed{\frac{3}{5} a^{8/3} - ab}$

d) $\int \dfrac{dx}{2x\sqrt{x^2 - 1}}$ Trig substitution: $x = \sec\theta \quad dx = \sec\theta \tan\theta \, d\theta$

$= \int \dfrac{\sec\theta \tan\theta \, d\theta}{2\sec\theta \sqrt{\sec^2\theta - 1}}$

$= \int \frac{1}{2} \, d\theta$

$= \frac{\theta}{2}$

$= \boxed{\dfrac{\operatorname{arcsec} x}{2} + C}$

11-23 $\lim\limits_{x\to 0} \dfrac{x-1+\sqrt{x+1}}{x} = \dfrac{0}{0}$ L'HOSPITAL!

$= \lim\limits_{x\to 0} \dfrac{1+\frac{1}{2}(x+1)^{-1/2}}{1} = \dfrac{3}{2}$ $\boxed{D}$

11-24 $f(x) = |x^{2/3} - 1| + 2$

has a vertex/corner where $x^{2/3} - 1 = 0$

$x = \pm 1$

Also, I can look for places where

$\frac{d}{dx}(x^{2/3} - 1) \neq \frac{d}{dx}(1 - x^{2/3})$

$\frac{2}{3}x^{-1/3} \neq -\frac{2}{3}x^{-1/3}$

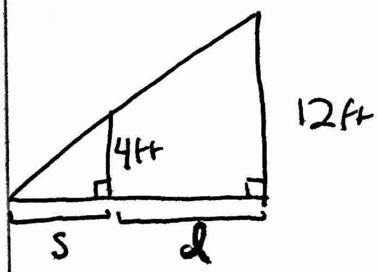
$x \neq 0$

$x = \{0, \pm 1\}$
E

11-25 $\frac{d}{dx}\int_{2x}^{x^2} (m^2 - 4m)\,dm = [(x^2)^2 - 4(x^2)]2x - [(2x)^2 - 4(2x)]2$

$= 2x^5 - 8x^3 - 8x^2 + 16x$ $\boxed{E}$

11-26



when $d = 6$, what is $\frac{ds}{dt}$?

if $\frac{dl}{dt} = 2$ ft/sec

$\frac{4}{s} = \frac{12}{s+d}$

$4s + 4d = 12s$

$4d = 8s$

$4\frac{dl}{dt} = 8\frac{ds}{dt}$ ← Note that this doesn't actually depend on l

$4(2) = 8\frac{ds}{dt}$

$1 = \frac{ds}{dt}$ 1 ft/sec $\boxed{A}$

You may use a calculator to verify the bounds of the definite integrals.

11-27 $2\int_0^{\pi/3} \frac{1}{2}(4\cos(\theta))^2 d\theta$ Should have 2 regions

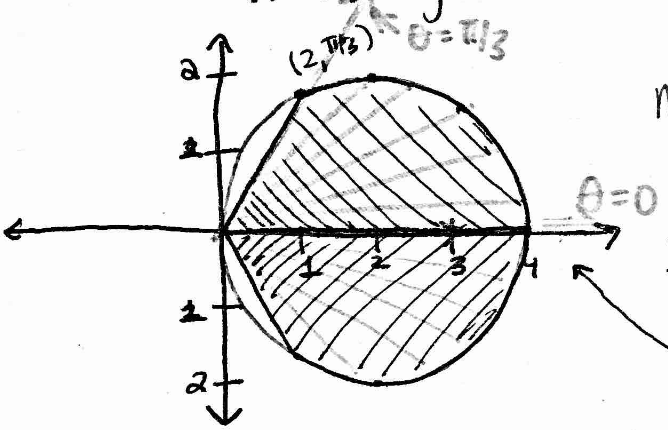
$r = 4\cos\theta$ ← That is the polar curve

$r^2 = 4r\cos\theta$

$x^2 + y^2 = 4x$

$(x^2 - 4x + 4) + y^2 = 4$

$(x-2)^2 + y^2 = 4$ ← circle w/ center at $(2,0)$ radius $= 2$



must be sectors w/ inner "corner" at the origin.

Investigate endpoints:

$\theta = 0$

$r = 4\cos(0) = 4$

$\theta = \pi/3$

$r = 4\cos(\pi/3) = 2$

It's the region shaded in blue, ~~bounded by~~ bounded by the circle $r = 4\cos\theta$ and the lines $\theta = \pi/3$ and $\theta = -\pi/3$

11-28 $r = 3$ and $r = 6\cos(\theta)$

$r = 3$

circle $x^2 + y^2 = 3$

center $(0,0)$

radius 3

$r = 6\cos\theta$

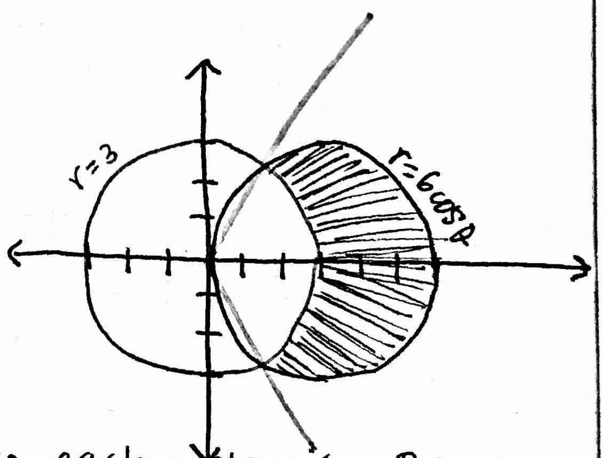
$r^2 = 6r\cos\theta$

$x^2 + y^2 = 6x$

$(x-3)^2 + y^2 = 9$

circle center $(3,0)$

radius 3



So, each sector is a BIG − SMALL sector, or "outside" − "inside"

→

11-28 continued...

First, figure out where the curves intersect

$r = r$

$3 = 6\cos(\theta)$

$1/2 = \cos(\theta)$

$\theta = \pi/3$ and $-\pi/3$

These will be the bounds of the integral

OUTER: INNER:

$$\int_{-\pi/3}^{\pi/3} \frac{1}{2}[6\cos\theta]^2 d\theta - \int_{-\pi/3}^{\pi/3} \frac{1}{2}[3]^2 d\theta$$

$$= 18\int_{-\pi/3}^{\pi/3} \cos^2\theta \, d\theta - \frac{9\theta}{2}\Big|_{-\pi/3}^{\pi/3}$$

$$= \frac{18}{2}\int_{-\pi/3}^{\pi/3} [\cos(2\theta)+1] d\theta - \frac{9\theta}{2}\Big|_{-\pi/3}^{\pi/3}$$

$$= 9\left[\frac{1}{2}\sin(2\theta)+\theta\right]\Big|_{-\pi/3}^{\pi/3} - \left[\frac{3\pi}{2} + \frac{3\pi}{2}\right]$$

$$= 9\left[\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right) + \frac{\pi}{3}\right] - 9\left[\frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right) - \frac{\pi}{3}\right] - 3\pi$$

$$= 9\left[\frac{\sqrt{3}}{2} + \frac{2\pi}{3}\right] - 3\pi$$

$$= \boxed{\frac{9\sqrt{3}}{2} + 3\pi \approx 17.219}$$

11-29 $\int_{-\pi/3}^{\pi/3} \frac{1}{2}[6\cos(\theta)^2 - 3^2] d\theta$ is correct b/c it is the difference btwn 2 complete sector areas.

$\int_{-\pi/3}^{\pi/3} \frac{1}{2}[6\cos\theta - 3]^2 d\theta$ doesn't make sense b/c it's a single sector created by subtracting the radii first.

11-30 Cardioid : $r = 2+2\cos\theta$ Circle : $r = 5\cos\theta$

a) on $-\pi \leq \theta \leq \pi$,

$$2+2\cos\theta = 5\cos\theta$$

$$\cos\theta = 2/3$$

$$\boxed{\begin{aligned}\theta &= \pm\arccos(2/3) \\ &\approx \pm 0.841\end{aligned}}$$

b) Circle is outer, cardioid is inner

$$\int_{-0.841}^{0.841} \frac{1}{2}\left[(5\cos\theta)^2 - (2+2\cos\theta)^2\right] d\theta$$

input into graphing calc...

$$\boxed{\approx 4.722 \text{ units}^2}$$

c) Inside cardioid − Inside circle

$$\int_{0.841}^{2\pi-0.841} \frac{1}{2}(2+2\cos\theta)^2 d\theta - 2\int_{0.841}^{\pi/2} \frac{1}{2}(5\cos\theta)^2 d\theta \approx 3.936$$

The area in part (b) is larger

— REVIEW + PREVIEW —

11-31 $x(t) = 20t$, $y(t) = 40t - 10t^2$

a) ball "lands" when $y(t) = 0$

$$40t - 10t^2 = 0$$
$$t = \{0, 4\}$$

4 seconds

b) How far does it travel horizontally? on $0 \leq t \leq 4$

$$0 \leq x(t) \leq 80$$

80 meters

→

11-31 c) How high does the ball go?

Max $y(t) = 40t - 10t^2$

$y'(t) = 40 - 20t$

$0 = 40 - 20t$

$t = 2$

@ $t=2$...

$y(2) = 40(2) - 10(2)^2$

$= 80 - 40$

$= \boxed{40 \text{ meters}}$

11-32 area within $r = 6 + 3\cos(5\theta)$

$\int_0^{2\pi} \frac{1}{2}\left[6 + 3\cos(5\theta)\right]^2 d\theta \approx \boxed{127.234 \text{ units}^2}$

11-33 a) $\int_2^6 \frac{dx}{\sqrt{x-2}}$ $u = \sqrt{x-2}$ $du = \frac{dx}{2\sqrt{x-2}}$

$dx = 2\sqrt{x-2}\, du$ $\int_2^6 \to \int_0^2$

$dx = 2u\,du$

$= \int_0^2 \frac{1}{u} \cdot 2u\,du$

$= \int_0^2 2\,du$

$= 2u\Big|_0^2$

$= \boxed{4}$

b) $\int \ln(x)\,dx$ parts $u = \ln x$ $dv = dx$

$du = \frac{1}{x}dx$ $v = 1x$

$= x\ln x - \int dx$

$= \boxed{x\ln x - x + C}$

c) $\int_3^{t^2} \frac{d}{dx}\left(\frac{x}{x-2}\right)dx = \left(\frac{t^2}{t^2-2}\right) - \frac{3}{3-2} = \frac{t^2 - 3t^2 + 6}{t^2-2} = \boxed{\frac{-2t^2+6}{t^2-2}}$

d) $\int_1^2 \frac{dx}{2x(x-3)}$ partial fractions $\rightarrow$

11-33d continued

$$\frac{1}{2x(x-3)} = \frac{A}{2x} + \frac{B}{x-3}$$

$$1 = A(x-3) + B(2x)$$

$x=3 \rightarrow 1 = 6B$ $\qquad$ $x=0 \rightarrow 1 = -3A$

$\frac{1}{6} = B$ $\qquad$ $-\frac{1}{3} = A$

$$\int_1^2 -\frac{1}{6x}\,dx + \int_1^2 \frac{1}{6(x-3)}\,dx$$

$$= \left[-\frac{1}{6}\ln|x| + \frac{1}{6}\ln|x-3|\right]\Big|_1^2$$

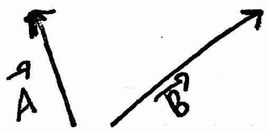
$$= -\frac{1}{6}\ln|2| + \frac{1}{6}\ln(1) - \left[-\frac{1}{6}\ln|1| + \frac{1}{6}\ln(2)\right]$$

$$= -\frac{1}{6}(\ln 2 + \ln 2)$$

$$= \boxed{-\frac{1}{6}\ln 4}$$

11-34 a) $\vec{A} = \langle -1, 3 \rangle = -i + 3j$

$\vec{B} = \langle 4, 3 \rangle = 4i + 3j$



b) for $\vec{A}$, $\theta = \tan^{-1}\left(\frac{3}{-1}\right) = -71°$, but $\vec{A}$ is in QII

so … $180 - 71° \approx \boxed{108°}$

For $\vec{B}$, $\theta = \tan^{-1}\left(\frac{3}{4}\right) \boxed{\approx 36.87°}$

c) $\vec{A} + \vec{B} = \boxed{\langle 3, 6 \rangle}$ $\qquad$ $\vec{B} - \vec{A} = \boxed{\langle 5, 0 \rangle}$

d) $\|\vec{B}\| \cdot \vec{A} = \sqrt{16+9} \cdot \langle -1, 3 \rangle$

$= 5 \cdot \langle -1, 3 \rangle$

$= \boxed{\langle -5, 15 \rangle}$

11-35 a) $f(x)=g(h(x))$

$\frac{dg}{dx}$ = change in g w/ respect to x

$\frac{dg}{dh}$ = change in g w/respect to h

$\frac{dh}{dx}$ = change in h w/ repect to x

b) $f(x)=g(h(x))$

$\frac{d}{dx}f(x) = g'(h(x)) \cdot h'(x)$

$\frac{dg}{dx} = \frac{dg}{dh} \cdot \frac{dh}{dx}$ ← chain rule

c) $\frac{dg}{dh} = \frac{dg}{dx} \div \frac{dh}{dx}$

11-36 $f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots = e^x$ $\boxed{B}$

11-37 The cubic $y=x^3-ax^2+2$ is concave up on...

$y' = 3x^2-2ax$

$y'' = 6x-2a$ ← when is y" positive?

$6x-2a>0$

$6x>2a$

$x>\frac{1}{3}a$ $(\frac{a}{3}, \infty)$ $\boxed{D}$

11-38 $a(t)=6t$

$v(t)=3t^2+C$, ~~[illegible]~~ $v(4)=50$

$50 = 48 + c$

$v(t)=3t^2+2$

$\int_0^9 v(t)dt$ gives displacement over 9 seconds

$\int_0^9 (3t^2+2)dt$

$= t^3+2t\Big|_0^9$

$= 729+18-0 = 747$ ft

+5 ft head start

$= 752$ ft $\boxed{E}$

11-39 $y = \sin^{-1}(x)$ $\frac{dx}{dt} = 4$, when $x = 1/2$, find $\frac{dy}{dt}$

$$\frac{d}{dt}(y) = \frac{d}{dt}\left(\sin^{-1}(x)\right)$$

$$\frac{dy}{dt} = \frac{1}{\sqrt{1-x^2}} \cdot \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{1}{\sqrt{1-(1/2)^2}} \cdot 4$$

$$= \frac{4}{\sqrt{3/4}} = \frac{4\sqrt{4}}{\sqrt{3}} = \frac{4\sqrt{12}}{3} = \frac{8\sqrt{3}}{3} \quad \boxed{B}$$