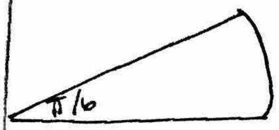
11-1



4.2cm

Area $= \frac{1}{12}$ circle $= \frac{1}{12}\pi(4.2)^2 \approx 4.618\ cm^2$

11-2 a) see resource page

b) see resource page

c) see resource page

11-3 If the sector angle is $\Delta\theta$, then the sector is $\frac{\Delta\theta}{2\pi}$ of a circle

$\therefore$ The area of the sector is

$$\frac{\Delta\theta}{2\pi}\pi R^2 = \frac{\Delta\theta}{2}R^2 = \frac{1}{2}R^2\Delta\theta$$

since the radius is determined by $f(\theta)$, $R = f(\theta)$

and the area is $\frac{1}{2}(f(\theta))^2\Delta\theta$

11-4 θ goes from a to b.

$$\int_a^b \frac{1}{2}(f(\theta))^2 d\theta$$

11-5 a) $R=5$

b) $\int_0^{2\pi}\frac{1}{2}(5)^2 d\theta = \frac{25}{2}(2\pi) - \frac{25}{2}(0) = \boxed{25\pi}$ wow!

11-6 $2\cos^2\theta - 1 = \cos(2\theta)$

$\cos^2\theta = \frac{1}{2}(\cos(2\theta)+1)$

since $\sin^2\theta = 1-\cos^2\theta$

$\sin^2\theta = 1 - \frac{1}{2}(\cos(2\theta)+1)$

a) $\int_0^{2\pi}\cos^2\theta\, d\theta$

$= \int_0^{2\pi}\frac{1}{2}[\cos(2\theta)+1]d\theta$

$= \frac{1}{2}\left[\frac{1}{2}\sin(2\theta)+\theta\right]\Big|_0^{2\pi}$

$= \frac{1}{2}[0+2\pi] - \frac{1}{2}[0+0]$

$= \boxed{\pi}$

b) $\int_0^{2\pi} 4\sin^2\theta\, d\theta$

$= 4\int_0^{2\pi}\left[1 - \frac{1}{2}\cos(2\theta) - \frac{1}{2}\right]$

$= 4\left[\frac{1}{2}\theta - \frac{1}{4}\sin(2\theta)\right]\Big|_0^{2\pi}$

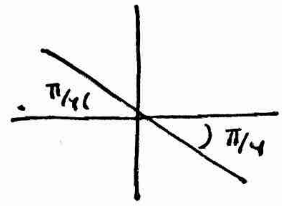
$= 4[\pi - 0] - 4[0-0]$

$= \boxed{4\pi}$

11-7 a) $r = \sin 3\theta + \cos 3\theta$ will cross the origin when $r = 0$

$0 = \sin 3\theta + \cos 3\theta$

$\sin 3\theta = -\cos 3\theta$ sine is opposite cosine @



$3\theta = \frac{3\pi}{4} + \pi k$

$\boxed{\theta = \frac{\pi}{4} + \frac{\pi}{3}k}$

b) $r = 3\sin^2\theta - 2$

$0 = 3\sin^2\theta - 2$

$3\sin^2\theta = 2$

$\sin^2\theta = 2/3$

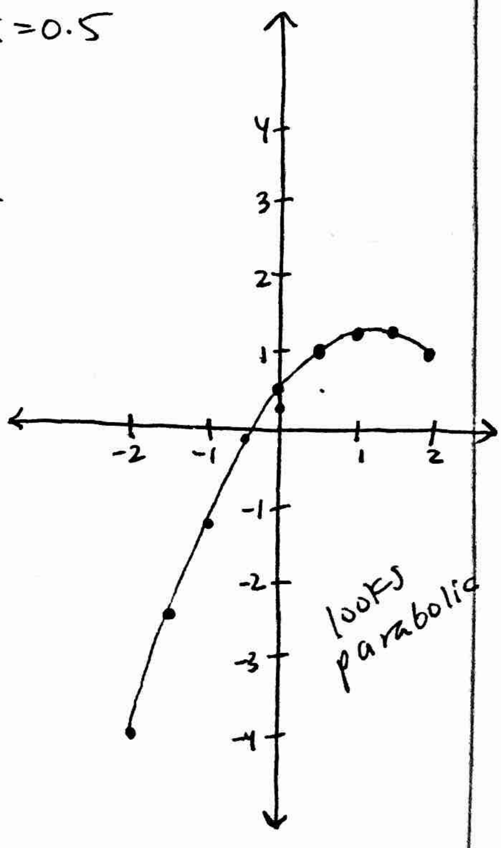
$\sin\theta = \pm\sqrt{2/3}$

$\theta = \arcsin(\sqrt{2/3}) + \pi k,\ \arcsin(-\sqrt{2/3}) + \pi k$

$\approx \pm 0.0955$ radians $+ \pi k$

11-8 $(-2, -4)$ $-2 \leq x \leq 2$ $dy/dx = 1 - x$ $\Delta x = 0.5$

x	y	slope	Next y
−2	−4	3	$-4 + 3(0.5) = -2.5$
−1.5	−2.5	2.5	$-2.5 + 2.5(0.5) = -1.25$
−1	−1.25	2	$-1.25 + 2(0.5) = -0.25$
−0.5	−0.25	1.5	$-0.25 + 1.5(0.5) = 0.5$
0	0.5	1	$0.5 + 1(0.5) = 1$
0.5	1	0.5	$1 + 0.5(0.5) = 1.25$
1	1.25	0	$1.25 + 0(0.5) = 1.25$
1.5	1.25	−0.5	$1.25 + (-0.5)(0.5) = 1$
2	1		



11-9 a) $\sum_{n=1}^{\infty} \frac{3}{5^n}$ converges. It's geometric w/ $|r|<1$

b) $\sum_{n=1}^{\infty} \frac{n!}{4^n}$ $\rho = \lim_{n\to\infty} \left| \frac{(n+1)!}{4^{n+1}} \cdot \frac{4^n}{n!} \right| = \lim_{n\to\infty} \left| \frac{n+1}{4} \right| = \infty$

diverges by the ratio test

c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3n-7}$ 1) $\left\{\frac{1}{3n-7}\right\}$ is positive & decreasing

$\lim_{n\to\infty} \frac{1}{3n-7} = 0$

converges by AST

d) $\sum_{n=1}^{\infty} \frac{1}{2}n^{-1} = \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n}$ is harmonic and diverges.

11-10 $\vec{a} = \langle -2, 5 \rangle$ $\vec{b} = \langle 6, 0 \rangle$ $\vec{c} = \langle \frac{3}{2}, 8 \rangle$

a) $2\vec{c} - \vec{a} = \langle \frac{3}{2}(2) + 2, 8(2) - 5 \rangle = \boxed{\langle 5, 11 \rangle}$

b) $-\vec{b} + \vec{c} = \langle -6 + 3/2, -0 + 8 \rangle = \boxed{\langle -\frac{9}{2}, 8 \rangle}$

c) $\|\vec{a} + \vec{b}\|$ = magnitude of $\vec{a} + \vec{b}$

$\vec{a} + \vec{b} = \langle 4, 5 \rangle$ magnitude $= \sqrt{16+25} = \boxed{\sqrt{41}}$

d) $\frac{\vec{b}}{\|\vec{b}\|} = \frac{\langle 6, 0 \rangle}{\sqrt{6^2+0^2}} = \frac{\langle 6, 0 \rangle}{6} = \boxed{\langle 1, 0 \rangle}$

11-11 $\begin{cases} x = 2t \\ y = t^2 \end{cases} \Rightarrow y = \left(\frac{x}{2}\right)^2$ $\begin{cases} x = t^2 \\ y = 2t \end{cases} \rightarrow x = \left(\frac{y}{2}\right)^2$

solve $\begin{cases} y = \left(\frac{x}{2}\right)^2 \\ x = \left(\frac{y}{2}\right)^2 \rightarrow y = 2\sqrt{x} \end{cases}$

$y = \left(\frac{x}{2}\right)^2$

$2\sqrt{x} = \left(\frac{x}{2}\right)^2$

$4x = \left(\frac{x^2}{4}\right)$

$x = \{0, 4\}$

$\boxed{\begin{matrix}(0,0)\\(4,4)\end{matrix}}$

11-12 $f(x)=\sin(x^2)$ $[0,3]$ find relative max & min

$$f'(x) = 2x\cos(x^2)$$

$$0 = 2x\cos(x^2)$$

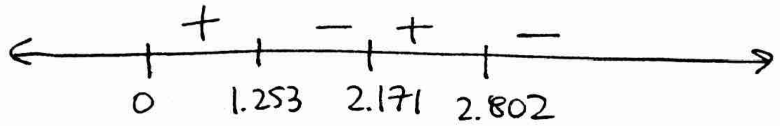
$2x=0$ $\qquad$ $\cos(x^2)=0$

$x=0$ $\qquad$ $x^2 = \frac{\pi}{2}+\pi k$

$$x = \pm\sqrt{\frac{\pi}{2}+\pi k}$$

on $[0,3]$, this includes $x \approx 2.171, 2.802, 1.253$

$f'(x)$



relative minimum @ $x=2.171$

relative maximum @ $x=1.253$ and $x=2.802$

11-13 This is Logistic. The rate $\frac{dP}{dt}$ is proportional (k) to the population who knows P and the pop. who doesn't $350k-P$

(A) $\frac{dP}{dt} = kP(350000-P)$

11-14 $f(x) = x + ax^{-2}$ $\qquad$ $f'(x) = 1 - 2ax^{-3}$

$$0 = 1 - 2ax^{-3}$$

$$ax^{-3} = \frac{1}{2}$$

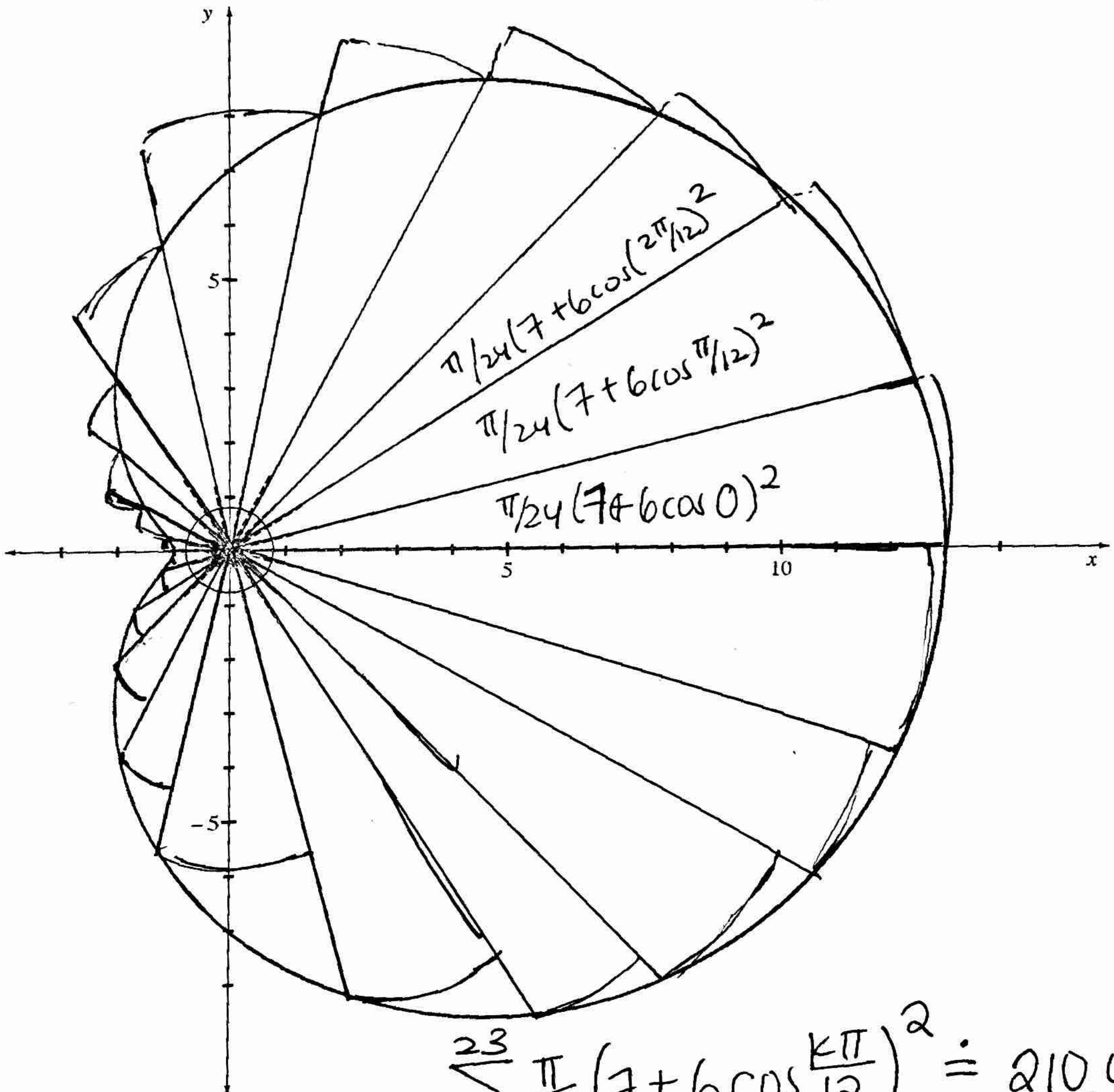
$$x^{-3} = \frac{1}{2a}$$

$$x^3 = 2a$$

$$x = \sqrt[3]{2a} \leftarrow \boxed{B}$$

Limaçon
$(r = 7 + 6\cos\theta)$

Sector area $= \frac{\pi}{24} R^2$
$= \frac{\pi}{24}(7 + 6\cos\theta)^2$



$$\sum_{K=0}^{23} \frac{\pi}{24}\left(7 + 6\cos\frac{K\pi}{12}\right)^2 \doteq \underline{210.487}$$